Exam 1

- Given the equations, find E_∞ and P_∞.

Chap 1

- be able to verify system properties. Show all your work for linearity, time invariance, ...
- Use the properties to relate the output to the input. (1.31)
- Sketch signals ($u(t)$, $u[n]$, $e^{-an}u[n]$, ...)

Chap 2

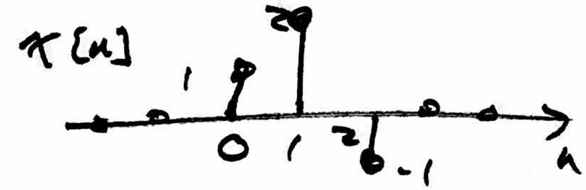
- Perform DT and CT convolution.
- Determine causality & stability from the impulse response (~~2.30, 2.~~ 2.28, 2.29)
- Find an impulse response given a system equation (2.30, 2.40)
- Find the output given an impulse response and input. (2.40, 2.31).

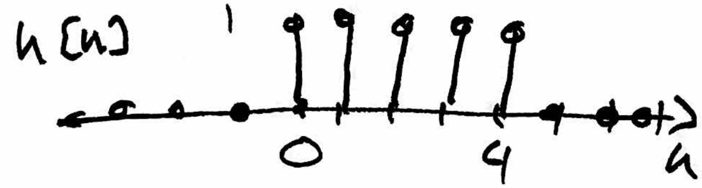
  not HW

Quiz 2

$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$$

$$h[n] = u[n] - u[n-5]$$





$$x[n] * h[n] = u[n] + 2u[n-1] - u[n-2]$$
$$- u[n-5] - 2u[n-6] + u[n+7]$$

Superposition

$$y[n] = h[n] + 2h[n-1] - h[n-2]$$

	1	1	1	1	1		
		2	2	2	2	2	
sum			-1	-1	-1	-1	-1

$$y[n] = \{\underline{1} \quad 3 \quad 2 \quad 2 \quad 2 \quad 1 \quad -1\}$$

$\uparrow$
$n=0$

Convolution table

				1	2	−1			
1	1	1	1	1					$= 1$
			…	1	1				$= 1 \cdot 1 + 2 \cdot 1 = 3$
			…	1	1	1			$= 1 + 2 - 1 = 2$
			…	1	1	1	1		$= 1 + 2 - 1 = 2$
				1	1	1	1	1	$= 1 + 2 - 1 = 2$
					1	1	1	1 1	$= 2 - 1 = 1$
						1	1	1 1 1	$= -1$

$$\Rightarrow y[n] = \{\underset{\substack{\uparrow \\ n=0}}{1}, 3, 2, 2, 2, 1, -1\}$$

$$x(t) = e^{-2t}u(t)$$
$$h(t) = u(t-1) - u(t-2)$$
$$y(t) = \int_{-\infty}^{t} x(\tau) f(\tau)\, d\tau$$

This example didn't go anywhere.

Go to 2.40

2.40

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau$$

$$x(t) = \delta(t) \Rightarrow y(t) = h(t)$$

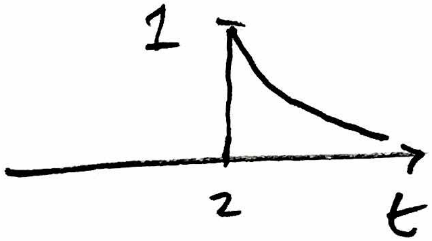
$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau - 2)\, d\tau$$

$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$$

sampling property

$$= \int_{-\infty}^{t} e^{-(t-2)} \delta(\tau - 2)\, d\tau$$

$$= e^{-(t-2)} \int_{-\infty}^{t} \delta(\tau - 2)\, d\tau = e^{-(t-2)} u(t-2)$$



$h(t)$ is causal because $u(t) = 0 \quad t < 0$

Stability
$$\int_{-\infty}^{\infty} \left| e^{-(t-2)} u(t-2) \right| dt = \int_{2}^{\infty} e^{-(t-2)} dt$$

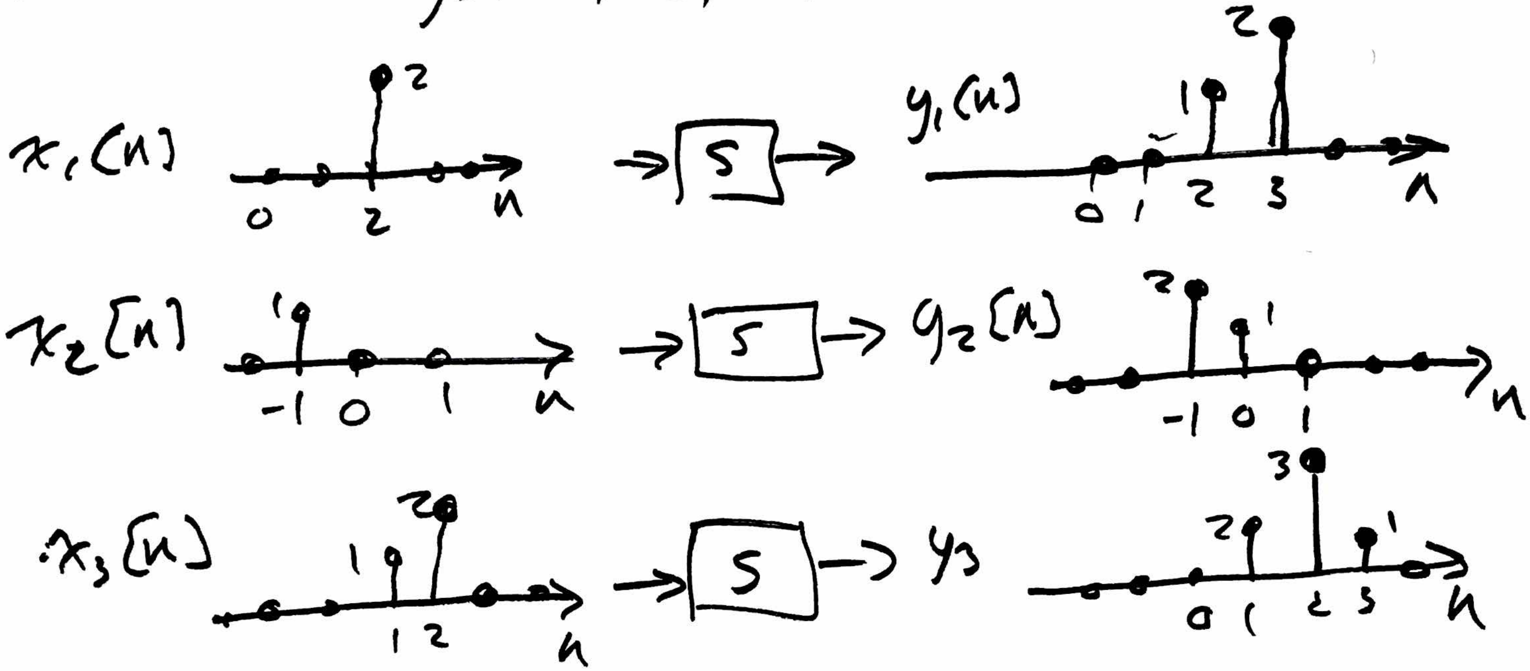
will be stable.

$$\int_{2}^{\infty} e^{-(t-2)} dt = -\left[e^{-(t-2)} \right]_{2}^{\infty}$$
$$= -\left[e^{-\infty} - e^{-(2-2)} \right]$$
$$= -[0-1]$$
$$= 1$$

$$\int_{-\infty}^{\infty} |h(t)| < \infty \Rightarrow \text{stable}$$

Ex Linearity & TI ⇒ Schaum's

There is a system, S, that is TI.



Is the system linear?

$$x_3[n] = x_1[n] + x_2[n-2]$$

$$S\{x_3[n]\} = S\{x_1[n]\} + S\{x_2[n-2]\}$$
$$= y_1[n] + y_2[n-2]$$

$y_1[n] + y_2[n-2]$ 

$\neq S\{x_3[n]\}$

not linear

Ex

$$y(t) = t\,x(t)$$

Linearity:

$$S\{a x_1(t) + b x_2(t)\} = t(a x_1(t) + b x_2(t))$$

$$= a\,t x_1(t) + b\,t x_2(t)$$

Linear

$$= a\,S\{x_1(t)\} + b\,S\{x_2(t)\}$$

Time invariance:

shifted output $y(t-t_0) = (t-t_0)\,x(t-t_0)$

shifted input $x_d(t) = x(t-t_0)$

$$y_d(t) = t\,x_d(t)$$

$$= t\,x(t-t_0)$$

$$\neq y(t-t_0)$$

Time varying

$$x_1(t) = \delta(t) \Rightarrow y_1(t) = 0\cdot\delta(t) = 0$$

$$x_2(t) = \delta(t-1) \Rightarrow y_2(t) = 1\cdot\delta(t-1)$$

Memory: $y(t)$ depends only on the current value of $x(t)$. $\Rightarrow$ memoryless.

Note: $y(t) = \cos(t-2) + x(t)$

$\Rightarrow$ still memoryless

Causal: memoryless $\rightarrow$ causal

Stability: say $x(t) = 1$

set Bound B

$y(B+1) = B+1$ $\rightarrow$ for any bound you can find a time where $y(t)$ exceeds it.

$x(t) = \frac{1}{t}$ $\quad y(t) = 1$

or, for $t \to \infty$ $y(t) \to \pm\infty$ for any finite $x(t)$ ~~?~~

"any" may be too strong