HOMEWORK-4

(1) $V(t) = i(t) \cdot R$

$$V_{RMS} = \sqrt{\frac{1}{T_2-T_1}\int_{T_1}^{T_2} |V(t)|^2 dt} = \sqrt{\frac{1}{T_2-T_1}\int_{T_1}^{T_2} |i(t)R|^2 dt}$$

$$= R\sqrt{\frac{1}{T_2-T_1}\int_{T_1}^{T_2} |i(t)|^2 dt} = R I_{RMS}$$

$$P_{avg}(t) = I(t)^2 R = I_{RMS}(I_{RMS}R)$$

$$= I_{RMS} V_{RMS}$$

2) (a) $P(X+N \leq y | X=1) = P(N \leq y-1) = F_N(y-1) \Rightarrow \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(y-1)^2}{2\sigma^2}\right)$

$= f_y(y|X=1)$

$P(\cancel{X}+N \leq y | X=-1) = P(N \leq y+1) = F_N(y+1)$

$\Rightarrow f_y(y|X=-1) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-(y+1)^2}{2\sigma^2}\right)$

(b) $f_y(y) = f_y(y|X=1)\, P(X=1) + f_y(y|X=-1)\, P(X=-1)$

$$f_y(y) = \frac{1/3}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(y-1)^2}{2\sigma^2}\right) + \frac{2/3}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(y+1)^2}{2\sigma^2}\right)$$

(c) $f_y(y|x=1)\,P(X=1) > f_y(y|x=-1)\,P(X=-1)$

$$\therefore \frac{1}{3}N(1,\sigma^2) > \frac{2}{3}N(-1,\sigma^2)$$

$$\frac{1}{3}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-(y-1)^2}{2\sigma^2}\right) > \frac{2}{3}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(y+1)^2}{2\sigma^2}\right)$$

$$\ln(1/3) + \frac{-(y-1)^2}{2\sigma^2} > \ln(2/3) + \frac{-(y+1)^2}{2\sigma^2}$$

$$\therefore y > \frac{\sigma^2 \ln(2)}{2}, T$$

(3) $P(\text{Heads}) = p \qquad P(\text{tails}) = 1-p$

$$P(\text{Coin flipped } k \text{ times}) = p^{k-1}(1-p) + (1-p)^{k-1}p$$

$$E[x] = E[x/H]\,P(H) + E[x/T]\,P(T) \quad \because \text{total expectation}$$

$$= \left[\sum_{k=2}^{\infty}(1-p)\,k\,p^{k-1}\right] \times p + \left[\sum_{k=2}^{\infty} k(1-p)^{k-1}p\right](1-p)$$

$$= (1-p)\left[\frac{-(p-2)}{(p-1)^2}\right] + \left[-\frac{(p-1)(p+1)}{(1-p)\,p}\right]$$

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(4) (a) $Y = U(X)\cdot X$

$$P(Y/R > 1) = 1 - \Phi(R/\sigma)$$

(b) $P(Y^2/R > 1) = P(Y^2 > R) = P(Y > \sqrt{R})$

$$= 1 - \Phi(\sqrt{R}/\sigma)$$

(c) $E[Y/R] = \frac{1}{R}E[Y] = \frac{2}{R}\int_0^\infty \frac{x}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-x^2}{2\sigma^2}\right)dx$

$$= \frac{2\sigma}{R\sqrt{2\pi}}$$

$E[Y^2/R^2] = \frac{1}{R^2}E[Y^2] = \frac{1}{R^2}\int_0^\infty x^2 \cdot 2f_X(x)dx = \frac{\sigma^2}{R^2}$

$\mathrm{Var}(Y/R) = \frac{1}{R^2}E[Y^2] - \frac{E[Y]^2}{R^2} = \frac{\sigma^2}{R^2} - \left(\frac{2\sigma}{R\sqrt{2\pi}}\right)^2$

(d) $E[Y^2/R] = \frac{1}{R}E[Y^2] = \frac{1}{R}\int_0^\infty x^2 \cdot \frac{2}{\sqrt{2\pi\sigma^2}}e^{-x^2/2\sigma^2}dx = \frac{\sigma^2}{R}$

$E[Y^4/R^2] = \frac{1}{R^2}E[Y^4] = \frac{1}{R^2}\int_0^\infty x^4 \frac{2}{\sqrt{2\pi\sigma^2}}e^{-x^2/2\sigma^2}dx = \frac{3\sigma^4}{R^{\cancel{4}}}$

$\mathrm{Var}[Y^2/R] = E[Y^4/R^2] - E[Y^2/R]^2$

$$= \frac{3\sigma^4}{R^2} - \left(\frac{\sigma^2}{R}\right)^2$$

(5) (a) $P(y \le y) = P(x_1 \le y, x_2 \le y, \cdots, x_n \le y)$

$$= P(x \le y)^n$$

$$= \frac{y^n}{a^n}$$

(b) $f_y(y) = \frac{d}{dy} P(y) = n \cdot \frac{y^{n-1}}{a^n}$

(c) $E[y] = \int_0^a y f_y(y) dy = \int_0^a y \cdot (n \frac{y^{n-1}}{a^n}) dy = \frac{na}{n+1}$

$$E[y^2] = \int_0^a y^2 (n \frac{y^{n-1}}{a^n}) dy = \frac{na^2}{n+2}$$

$$var(y) = E[y^2] - E[y]^2 = a^2 \left[\frac{n}{n+2} - \left(\frac{n^2}{n+1^2}\right)\right]$$

y is a good estimate for 'a' as $\lim_{n \to \infty} E[y] = \lim_{n \to a} \frac{na}{n+1} = a$

(6)(a) $f_X(x) = \frac{1}{2a}$

$$E[e^{j\omega x}] = \int_{-a}^{a} \frac{1}{2a} e^{j\omega x} dx = \frac{e^{j\omega a} - e^{-j\omega a}}{2aj\omega}$$

(b) $\eta_X = 1/\lambda, \lambda > 0$

$$\Psi_X(\omega) = \int_{-\infty}^{\infty} \lambda e^{-\lambda x} u(x) e^{j\omega x} dx \Rightarrow \frac{\lambda}{\lambda - j\omega} \Phi_X(\omega)$$

(c) $E[x^n] = \frac{1}{j^n} \frac{d^n}{d\omega^n} \Phi_X(\omega) \Big|_{\omega=0}$

$$= \frac{1}{j^n} \cdot \frac{d^n}{d\omega^n} \lambda(\lambda - j\omega)^{-1} \Big|_{\omega=0}$$

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