(1) a) $x_1(t) = \frac{\sin 2\pi t}{t} = \pi \cdot \frac{\sin 2\pi t}{\pi t} \xrightarrow{\mathcal{F}} \pi\left[u(\omega+2\pi) - u(\omega-2\pi)\right]$ (19)

when $|\omega| > 2\pi$, $X_1(\omega) = \phi$, $\therefore$ This is $\boxed{\text{band-limited}}$.

$\omega_N = \boxed{4\pi}$

b) $x_2(t) = 3\left[u[t+2] - u[t-2]\right] \xrightarrow{\mathcal{F}} \frac{6\sin 2\omega}{\omega}$ (21)

Because of the sharp corners and $X_2(\omega)$ which is not ϕ for some $\omega > \omega_c$, this is $\boxed{\text{not band-limited}}$

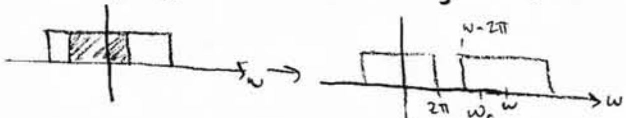
c) $x_3(t) = \frac{\sin(2\pi t)}{t} \sum_{n=-\infty}^{\infty} \delta(t - \frac{n}{2})$

$$X_3(\omega) = \int_{-\infty}^{\infty} \underbrace{\frac{\sin(2\pi t)}{t} \sum_{n=-\infty}^{\infty} \delta(t-\tfrac{n}{2})}_{} e^{-j\omega t}\, dt$$

$$= \sum_{n=-\infty}^{\infty} \frac{\sin(2\pi \frac{n}{2})}{n/2} e^{-j\omega n/2} = \underbrace{\sum_{n=-\infty}^{\infty} \frac{2\sin(\pi n)}{n} e^{-j\omega n/2}}_{}.$$

we see that there is no $\omega > \omega_c$ s.t. $X_3(\omega)$ is ϕ, therefore this is $\boxed{\text{not band-limited}}$

d) $x_4(t) = \left(\frac{\sin(2\pi t)}{t}\right)^2 \xrightarrow{\mathcal{F}} \frac{1}{2\pi}\pi\left[u(\omega+2\pi) - u[\omega-2\pi)\right] * \pi\left[u(\omega+2\pi) - u(\omega-2\pi)\right].$ (14)

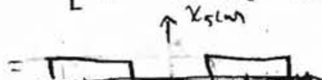


$\omega_c = 4\pi$

$\omega_N = \boxed{8\pi}$

we can see illustrated the convolution of X_4 with itself. The shaded area represents the value of the convolution. It is shown that after $|\omega| > \omega_c$ $X_4(\omega) = \phi$, thus $\boxed{\text{band limited}}$

e) $x_5(t) = \frac{\sin(2\pi t)}{t} \cdot \cos(5\pi t) \xrightarrow{\mathcal{F}} \frac{1}{2}\left[u(\omega+2\pi) - u(\omega-2\pi)\right] * \frac{1}{2}\left[\delta(\omega+5\pi) + \delta(\omega-5\pi)\right]$



Thus for all $|\omega| > 7\pi$, $X_5(\omega) = 0$

$\boxed{\text{Band Limited}}$ $\omega_N = \boxed{14\pi}$

f) $x_6(t) = \frac{\sin 2\pi t}{t} \cdot e^{j5\pi t} \xrightarrow{\mathcal{F}} \pi\left[u(\omega+2\pi-5\pi) - u[\omega-2\pi-5\pi)\right]$ (19, 12)

$= \pi\left[u(\omega-3\pi) - u(\omega-7\pi)\right]$

Thus for all $|\omega| > 7\pi$, $X_6(\omega) = \phi$ $\therefore$ $\boxed{\text{Band-Limited}}$

$\omega_N = \boxed{14\pi}$

(2) $x[n] = 3^n u[-n-1]$ $\qquad X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$ (55)

$$X(z) = \sum_{n=-\infty}^{\infty} 3^n \underbrace{u[-n-1]}\, z^{-n}$$

when $-n-1 \geq 0$, this is one

$$u[\square] = \begin{cases} 1, & n \leq -1 \\ 0, & \text{else} \end{cases}$$

$$= \sum_{n=-\infty}^{-1} 3^n z^{-n}$$

Let $m = -n$

$$= \sum_{m=\infty}^{1} 3^{-m} z^{m}$$

$$= \sum_{m=1}^{\infty} \left(\frac{z}{3}\right)^m$$

$$= \frac{z}{3} \sum_{m=0}^{\infty} \left(\frac{z}{3}\right)^m = \frac{z}{3} \frac{1}{1-z/3}, \quad |z| < 3$$

$$= \frac{\cancel{3}z}{\cancel{3}(3-z)}, \quad |z| < 3$$

$$= \frac{1}{3/z - 1}, \quad |z| < 3$$

$$= \boxed{\frac{-1}{1-3z^{-1}}, \quad |z| < 3} \checkmark$$

$$-3^n u[-n-1] \xrightarrow{Z} \frac{1}{1-3z^{-1}}$$

$|z| < 3$ (65)

$$\therefore X(z) = \frac{-1}{1-3z^{-1}}$$

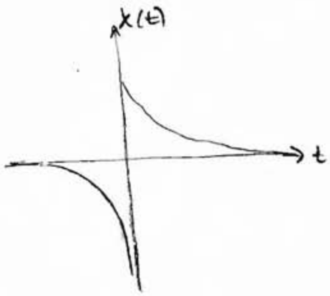
$|z| < 3$

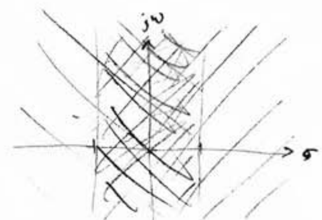
(3) $H(s) = \frac{1}{s+2}$, $Re(s) > -2$

Find $y(t)$ when

$$x(t) = \begin{cases} e^{-3t}, & t>0 \\ 0, & t=0 \\ -e^{3t}, & t<0 \end{cases}$$

Because we do not know if the system is LTI, we cannot answer this question.





$$x(t) = -e^{3t}u(-t) + e^{-3t}u(t)$$

$$X(s) = \begin{cases} \frac{1}{s-3} & Re(s) < 3 \quad + \\ \frac{1}{s+3} & Re(s) > -3 \end{cases}$$

$h(t) = e^{-2t}u(t)$

- Assume LTI

$$Y(s) = X(s) \cdot H(s)$$

$$= \frac{1}{s-3} \cdot \frac{1}{s+2} + \frac{1}{s+3} \cdot \frac{1}{s+2}, \quad -3 < Re[s] < 3$$

$$= \frac{1}{5} \cdot \underset{a=-3}{\frac{1}{s-3}} - \frac{1}{5} \cdot \underset{a=-2}{\frac{1}{s+2}} + \underset{a=-3}{\frac{1}{s+3}} - \underset{a=-2}{\frac{1}{s+2}}, \quad -3 < Re[s] < 3$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$A(s+2) + B(s-3) = 1$

$s=-2, \; B = -\frac{1}{5}$

$s=3, \; A = \frac{1}{5}$

$C(s+2) + D(s+3) = 1$

$s=-2, \; D = 1$

$s=-3, \; C = -1$

$$y(t) = -\frac{1}{5}e^{3t}u(-t) - \frac{1}{5}e^{2t}u(t) - e^{3t}u(-t) - e^{2t}u(t)$$

$$= \boxed{-\frac{6}{5}e^{3t}u(-t) - \frac{6}{5}e^{2t}u(t)}$$

(4) $\frac{dy_c(t)}{dt} + y_c(t) = x_c(t)$, $x_c(t) = \delta(t)$

a) $\mathcal{L}\left[\text{"} \qquad \text{"}\right] = \mathcal{L}\left[\text{"}\ \text{"}\right]$

$$sY_c(s) + Y_c(s) = X_c(s)$$

$$Y_c(s) = \frac{X_c(s)}{s+1} = X_c(s)H(s)$$

$$H(s) = \frac{1}{s+1} \xrightarrow{\mathcal{L}^{-1}} \pm e^{-t}u(\pm t) \quad \left[\text{Based on ROC}\right]$$

so $y_c(t) = x_c(t) * h(t)$ (45)

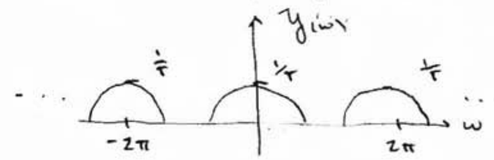
$$= \int_{-\infty}^{\infty} h(t-\tau)\delta(\tau)\,d\tau$$

$= h(t) = e^{-t}u(t)$ because this is causal, $h(t) = \phi,\ t<0$

b) $w[n] = \delta[n]$
Find $H(\omega)$, $h[n]$

$$y[n] \rightarrow \boxed{H(\omega)} \rightarrow w[n] = \delta[n]$$

$y[n]$ is sampling of $y_c(t) = e^{-t}u(t)$



$$H(\omega) = \frac{W(\omega)}{Y(\omega)} = \frac{1}{Y(\omega)}$$

$$= \boxed{1 - e^{-j\omega T}}$$

(36) (24,36)

$$\Big\downarrow \mathcal{F}^{-1}$$

$$\boxed{h[n] = \delta[n] - e^{-T}\delta[n-1]}$$

$$y[n] = y_c(nT) = e^{-nT}u(nT)$$

$\alpha^n u[n],\ |\alpha|<1 \xrightarrow{\mathcal{F}} \frac{1}{1-\alpha e^{-j\omega}}$ (38)

$$\alpha = e^{-T}$$

$$y[n] = (e^{-T})^n u[n]$$

$$Y(\omega) = \frac{1}{1 - e^{-T}e^{-j\omega}}$$