ECE 301

Division 1, Fall 2008

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Final Examination

Instructions:

1. Wait for the “BEGIN” signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
3. At the end of this document is a 5 page table of formulas and 4 pages of scratch paper. You may detach these **once the exam begins** provided you sign your name on top of each page and slide them back inside your exam before handing it in.
4. This is a closed book exam. The only personal items allowed are pens/pencils, erasers, your Purdue ID and something to drink. Anything else is strictly forbidden.
5. You must keep your eyes on your exam at all times. Looking around is strictly forbidden.
6. Please leave your Purdue ID out so the proctors may check your identity.

Name: ________________________________

Email: ________________________________

Signature: ____________________________

Itemized Scores	
Problem 1:	Problem 6:
Problem 2:	Problem 7:
Problem 3:	Problem 8:
Problem 4:	
Problem 5:	
Total:	

(10 pts) **1.** Compute the energy and the power of the signal

$$x[n] = \frac{1+3j}{e^{\sqrt{2}\pi j n}}.$$

(20 pts) **2.** NOTE: EVEN THOUGH THIS PAGE CONTAINS MULTIPLE CHOICE QUESTIONS, IT WOULD BE UNWISE TO ATTEMPT TO CHEAT, AS THE QUESTIONS ARE PERMUTED IN THE 4 DIFFERENT VERSIONS OF THIS TEST.

a) Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no. (No justification needed).

	Yes	No
If $y(t) = (t+5)x(t)$, is the system causal?	☐	☐
If $y(t) = x(t^2)$, is the system causal?	☐	☐
If $y(t) = x(5-t)$, is the system memoryless?	☐	☐
If $y(t) = x(t/3)$, is the system stable?	☐	☐
If $y(t) = x(t/3)$, is the system linear?	☐	☐
If $y(t) = u(t) * x(t)$, is the system LTI?	☐	☐

b) Are the following statements true or false? (No justification needed.)

	True	False
In a cascade of a time-scaling and a time-delay, the order of the systems does not matter.	☐	☐
In a cascade of LTI systems, the order of the systems does not matter.	☐	☐
If two linear systems have the same unit impulse response, then the two systems are the same.	☐	☐
$e^{2\pi jt} = \left(e^{2\pi j}\right)^t = 1^t = 1.$	☐	☐

(25 pts) **3.** You have learned that a discrete-time LTI system can be specified in several ways, such as

1. an explicit input-output formula $y[n] = f\,(x[n], n)$,
2. the unit impulse response of the system $h[n]$,
3. the system's function $H(z)$, with its ROC,
4. a difference equation.

For each system below, you are given one of these representations. Find the others. (No justification needed.)

	System 1	System 2
Explicit Input-output Formula	$y[n] = \sum_{k=-\infty}^{n} x(k)$	
$h[n]$		$3^n u[-n]$
$H(z)$, ROC		
Difference Equation		

(5 pts) **4.** The Laplace transform of the unit impulse response of an LTI system is $H(s) = \frac{s+3}{s^2+s-4}$. What is the system's response to the input $x(t) = e^{2t}$ (No justification needed.)

(15 pts) **5.** Let $x(t)$ be a periodic signal with fundamental frequency ω_0. Without using the table of Fourier transforms pairs, prove that the Fourier transform of $x(t)$ is

$$\mathcal{X}(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta\left(\omega - k\omega_0\right)$$

where the a_k's are the Fourier series coefficients of $x(t)$. Note: if you write nothing on this page, you will automatically get 3 pts.

(15 pts) **6.** Obtain the inverse Laplace transform of

$$X(s) = \frac{1}{s^2 + 3s + 2}, \qquad \text{ROC: } \operatorname{Re}(s) < -2.$$

Note: if you write nothing on this page, you will automatically get 3 pts.

(15 pts) **7.** Using the definition of the z-transform (and without otherwise using the table), compute the inverse z-transform of

$$X(z) = \frac{z}{1 + \frac{1}{4}z}, |z| > 4.$$

Note: if you write nothing on this page, you will automatically get 3 pts.

(25 pts) **8.** Let $x(t)$ be a pure imaginary and even signal with Nyquist rate equal to π.

(3+2pts) a) Sketch the graph of a Fourier transform that *could* be the Fourier transform of $x(t)$. (You will get two extra points if it has a cute shape!) Then base your answer to b) and c) on this graph.

(10 pts) b) Sketch the graph of the Fourier transform of $x_1(t) = x(t) \sum_{k=-\infty}^{\infty} \delta\left(t - \frac{2k}{3}\right)$.

(10 pts) c) Sketch the graph of the Fourier transform of $x_2[n] = x(\frac{2}{3}n)$.

Table

DT Signal Energy and Power

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 \tag{1}$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \tag{2}$$

CT Signal Energy and Power

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \tag{3}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \tag{4}$$

Fourier Series of CT Periodic Signals with period T

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t} \tag{5}$$

$$a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\left(\frac{2\pi}{T}\right)t} dt \tag{6}$$

Fourier Series of DT Periodic Signals with period N

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\left(\frac{2\pi}{N}\right)n} \tag{7}$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\left(\frac{2\pi}{N}\right)n} \tag{8}$$

CT Fourier Transform

$$\text{F.T. : } \mathcal{X}(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \tag{9}$$

$$\text{Inverse F.T.: } x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{X}(\omega)e^{j\omega t}d\omega \tag{10}$$

Properties of CT Fourier Transform

Let $x(t)$ be a continuous-time signal and denote by $\mathcal{X}(\omega)$ its Fourier transform. Let $y(t)$ be another continuous-time signal and denote by $\mathcal{Y}(\omega)$ its Fourier transform.

	Signal	*FT*	
Linearity:	$ax(t) + by(t)$	$a\mathcal{X}(\omega) + b\mathcal{Y}(\omega)$	(11)
Time Shifting:	$x(t - t_0)$	$e^{-j\omega t_0}\mathcal{X}(\omega)$	(12)
Frequency Shifting:	$e^{j\omega_0 t}x(t)$	$\mathcal{X}(\omega - \omega_0)$	(13)
Time and Frequency Scaling:	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	(14)
Multiplication:	$x(t)y(t)$	$\frac{1}{2\pi}\mathcal{X}(\omega) * \mathcal{Y}(\omega)$	(15)
Convolution:	$x(t) * y(t)$	$\mathcal{X}(\omega)\mathcal{Y}(\omega)$	(16)
Differentiation in Time:	$\frac{d}{dt}x(t)$	$j\omega\mathcal{X}(\omega)$	(17)

Some CT Fourier Transform Pairs

$$e^{j\omega_0 t} \xrightarrow{\mathcal{F}} 2\pi\delta(\omega - \omega_0) \tag{18}$$

$$1 \xrightarrow{\mathcal{F}} 2\pi\delta(\omega) \tag{19}$$

$$\frac{\sin Wt}{\pi t} \xrightarrow{\mathcal{F}} u(\omega + W) - u(\omega - W) \tag{20}$$

$$u(t + T_1) - u(t - T_1) \xrightarrow{\mathcal{F}} \frac{2\sin(\omega T_1)}{\omega} \tag{21}$$

$$\delta(t) \xrightarrow{\mathcal{F}} 1 \tag{22}$$

$$e^{-at}u(t), \mathcal{R}e\{a\} > 0 \xrightarrow{\mathcal{F}} \frac{1}{a + j\omega} \tag{23}$$

$$te^{-at}u(t), \mathcal{R}e\{a\} > 0 \xrightarrow{\mathcal{F}} \frac{1}{(a + j\omega)^2} \tag{24}$$

DT Fourier Transform

Let $x[n]$ be a discrete-time signal and denote by $X(\omega)$ its Fourier transform.

$$\text{F.T.:}\mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \tag{25}$$

$$\text{Inverse F.T.: } x[n] = \frac{1}{2\pi}\int_{2\pi} \mathcal{X}(\omega)e^{j\omega n}d\omega \tag{26}$$

Properties of DT Fourier Transform

Let $x(t)$ be a signal and denote by $\mathcal{X}(\omega)$ its Fourier transform. Let $y(t)$ be another signal and denote by $\mathcal{Y}(\omega)$ its Fourier transform.

	Signal	*F.T.*	
Linearity:	$ax[n] + by[n]$	$a\mathcal{X}(\omega) + b\mathcal{Y}(\omega)$	(27)
Time Shifting:	$x[n - n_0]$	$e^{-j\omega n_0}\mathcal{X}(\omega)$	(28)
Frequency Shifting:	$e^{j\omega_0 n}x[n]$	$\mathcal{X}(\omega - \omega_0)$	(29)
Time Reversal:	$x[-n]$	$\mathcal{X}(-\omega)$	(30)
Multiplication:	$x[n]y[n]$	$\frac{1}{2\pi}\mathcal{X}(\omega) * \mathcal{Y}(\omega)$	(31)
Convolution:	$x[n] * y[n]$	$\mathcal{X}(\omega)\mathcal{Y}(\omega)$	(32)
Differencing in Time:	$x[n] - x[n-1]$	$(1 - e^{-j\omega})\mathcal{X}(\omega)$	(33)

Some DT Fourier Transform Pairs

$$\sum_{k=0}^{N-1} a_k e^{jk(\frac{2\pi}{N})n} \xrightarrow{\mathcal{F}} 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - \frac{2\pi k}{N}) \tag{34}$$

$$e^{j\omega_0 n} \xrightarrow{\mathcal{F}} 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l) \tag{35}$$

$$1 \xrightarrow{\mathcal{F}} 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l) \tag{36}$$

$$\frac{\sin Wn}{\pi n}, 0 < W < \pi \xrightarrow{\mathcal{F}} \mathcal{X}(\omega) = \begin{cases} 1, & 0 \leq |\omega| < W \\ 0, & \pi \geq |\omega| > W \end{cases} \tag{37}$$

$$\mathcal{X}(\omega)\text{periodic with period } 2\pi$$

$$\delta[n] \xrightarrow{\mathcal{F}} 1 \tag{38}$$

$$u[n] \xrightarrow{\mathcal{F}} \frac{1}{1 - e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k) \tag{39}$$

$$\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{1 - \alpha e^{-j\omega}} \tag{40}$$

$$(n+1)\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{(1 - \alpha e^{-j\omega})^2} \tag{41}$$

Laplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \tag{42}$$

Properties of Laplace Transform

Let $x(t)$, $x_1(t)$ and $x_2(t)$ be three CT signals and denote by $X(s)$, $X_1(s)$ and $X_2(s)$ their respective Laplace transform. Let R be the ROC of $X(s)$, let R_1 be the ROC of $X_1(z)$ and let R_2 be the ROC of $X_2(s)$.

	Signal	L.T.	ROC	
Linearity:	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$	(43)
Time Shifting:	$x(t - t_0)$	$e^{-st_0}X(s)$	R	(44)
Shifting in s:	$e^{s_0 t}x(t)$	$X(s - s_0)$	$R + s_0$	(45)
Conjugation:	$x^*(t)$	$X^*(s^*)$	R	(46)
Time Scaling:	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	aR	(47)
Convolution:	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$	(48)
Differentiation in Time:	$\frac{d}{dt}x(t)$	$sX(s)$	At least R	(49)
Differentiation in s:	$-tx(t)$	$\frac{dX(s)}{ds}$	R	(50)
Integration :	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \mathcal{R}e\{s\} > 0$	(51)

Some Laplace Transform Pairs

Signal	*LT*	*ROC*	
$\delta(t)$	1	all s	(52)
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\mathcal{R}e\{s\} > -\alpha$	(53)
$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\mathcal{R}e\{s\} < -\alpha$	(54)

z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \quad (55)$$

Properties of z-Transform

Let $x[n]$, $x_1[n]$ and $x_2[n]$ be three DT signals and denote by $X(z)$, $X_1(z)$ and $X_2(z)$ their respective z-transform. Let R be the ROC of $X(z)$, let R_1 be the ROC of $X_1(z)$ and let R_2 be the ROC of $X_2(z)$.

	Signal	z-T.	ROC	
Linearity:	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least $R_1 \cap R_2$	(56)
Time Shifting:	$x[n - n_0]$	$z^{-n_0}X(z)$	R, but perhaps adding/deleting $z = 0$	(57)
Time Shifting:	$x[-n]$	$X(z^{-1})$	R^{-1}	(58)
Scaling in z:	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	R	(59)
Conjugation:	$x^*[n]$	$X^*(z^*)$	R	(60)
Convolution:	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least $R_1 \cap R_2$	(61)

Some z-Transform Pairs

Signal	LT	ROC	
$u[n]$	$\frac{1}{1 - z^{-1}}$	$\|z\| > 1$	(62)
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$\|z\| < 1$	(63)
$\alpha^n u[n]$	$\frac{1}{1 - \alpha z^{-1}}$	$\|z\| > \alpha$	(64)
$-\alpha^n u[-n - 1]$	$\frac{1}{1 - \alpha z^{-1}}$	$\|z\| < \alpha$	(65)
$\delta[n]$	1	all z	(66)

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