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3:02 PM

Example of Possible Questions

Suppose a DT signal $x[n]$ satisfies...

1. $x[n]$ is periodic $N=6$
2. $\sum_{n=0}^{5} x[n] = 2$
3. $\sum_{n=2}^{7} (-1)^n x[n] = 1$
4. $x[n]$ has minimum power per period among the set of signals that satisfy 1, 2, & 3.

Soln: by 1 $\quad x[n] = \sum_{k=0}^{5} a_k e^{-jk\frac{\pi}{3}n}$

by 2: $\quad a_0$ = average over one period $= \frac{1}{6}\sum_{n=0}^{5} x[n] = \frac{1}{6}\cdot 2 = \frac{1}{3}$

by 3: $\quad$ observe $e^{-jk\frac{\pi}{3}n} = (-1)^n$ when $k=3$

$$a_3 = \frac{1}{6}\sum_{n=0}^{5} x[n]\, e^{-j\pi n} = \frac{1}{6}\sum_{n=0}^{5} x[n](-1)^n$$

but $x[n+6] = x[n]$ and $(-1)^{n+6} = (-1)^n$ so $=$

$$= \frac{1}{6}\sum_{n=2}^{7} x[n](-1)^n = \frac{1}{6}(1) = \frac{1}{6} = a_3$$

so far we have

$$x[n] = \frac{1}{3} + a_1 e^{-j\frac{\pi}{3}n} + a_2 e^{-j\frac{2\pi}{3}n} + \frac{1}{6}e^{-j\pi n} + a_4 e^{-j\frac{4\pi}{3}n} + a_5 e^{-j\frac{5\pi}{3}n}$$

power over one period is $\frac{1}{6}\sum_{n=0}^{5} |x[n]|^2 = \sum_{n=0}^{5} |a_k|$

from 4, we want minimum power

so $\sum_{n=0}^{5} \frac{1}{3}^2 + |a_1|^2 + |a_2|^2 + (\frac{1}{6})^2 + |a_4|^2 + |a_5|^2$

to get smallest, let $a_1, a_2, a_4, a_5 = 0$

$$\boxed{\text{So } x[n] = \frac{1}{3} + \frac{1}{6}e^{j\pi n}}$$