1. Let \( g \in C^1(\mathbb{R}) \). Show \( g \) takes sets of measure zero to sets of measure zero.

2. If \( E_1, E_2 \) are measurable sets in \( \mathbb{R} \), show \( E_1 \times E_2 \) is measurable in \( \mathbb{R}^2 \).

3. Let \( f \) be continuous on \([1, \infty)\) and \( \lim_{x \to \infty} f(x) = 0 \). Suppose that \( \int_1^\infty f(x)x^n dx = 0 \) for \( n = -2, -3, \ldots \). Does it follow that \( f \equiv 0 \)?

4. Given \((X, \mathcal{M}, \mu), \mu(X) < \infty\), and let \( f_n \to f \) pointwise on \( X \), \( f_n : X \to \mathbb{R} \) measurable. Assume that for each \( \epsilon > 0 \), there is a \( \delta = \delta(\epsilon) \) such that \( E \in \mathcal{M} \) and \( \mu(E) \leq \delta \) implies that \( |\int_E f_n d\mu| \leq \epsilon \). Show that \( f_n \to f \) in \( L^1 \).

5. Let \((X, \mathcal{M}, \mu)\) be a measure space with \( \mu(X) = 1 \). Let \( E_j \in \mathcal{M}, j = 1, \ldots, n \). If every \( x \in X \) is in at least \( k \) of these sets, show that there exists \( 1 \leq j_0 \leq n \) such that \( \mu(E_{j_0}) \geq \frac{k}{n} \).

6. Let \( f \in L^1(I_0), f \geq 0 \), and let for each positive integer \( n \),

\[
f_n(x) = \begin{cases} n, & f(x) \geq n \\ f(x), & f(x) < n \end{cases}.
\]

Show that

\[
\int_0^1 \log f_n \, dx \to \int_0^1 \log f \, dx.
\]

Note that the integrals could be \(-\infty\).

7. Let \( A \subset \mathbb{R} \), \( A \) measurable, and show that \( \forall r \in [0, |A|], \exists E \subset A \) with \( |E| = r \). Can it be generalized to \( \mathbb{R}^n \)?

8. Evaluate the following limits and fully justify your answers:

(a) \( \lim_{n \to \infty} \int_0^n \left(1 - \frac{x}{n}\right)^n e^{x/2} \, dx \)

(b) \( \lim_{n \to \infty} \int_0^\infty \frac{n}{e^x + n^2x} \, dx \)
9. Let \( \{f_n\} \) be a sequence of nonnegative functions in \( L^1([0, 1]) \) with the property that

\[
\int_0^1 f_n(t)\,dt = 1 \quad \text{and} \quad \int_{1/n}^1 f_n(t)\,dt \leq \frac{1}{n}
\]

for all \( n \). Define \( h(x) = \sup_n f_n(x) \). Prove that \( h \not\in L^1([0, 1]) \).

10. Let \( 1 > \epsilon_j > 0, j = 1, 2, \ldots \). Show that \( \sum \epsilon_j < \infty \) is necessary and sufficient so that \( \sum \chi_{A_j}(x) < \infty \) a.e. whenever \( \{A_j\} \) is a sequence of Borel sets in \( I_0 \) with \( m(A_j) = \epsilon_j \).