EE 301 **Final Exam** **12/14/04**

Name: ______________________

General Instructions:

- The exam is closed book and closed notes. Calculators are **not** allowed or needed.
- A formula sheet will be handed out.
- Your work must be explained to receive full credit.
- Point values for each problem are as indicated. The exam totals 108 points (a score of 100 or above will be counted as 100).
- All plots must be carefully drawn with axes labeled.
- If you finish the exam during the first 110 minutes, you may turn it in and leave. During the last 10 minutes you must remain seated until we pick up exams from everyone.

This exam is for Krogmeier's section of 301.

Do not open the exam until you are told to begin.

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Problem 1. [18 pts. total] Short answer. The following sub-problems may be solved independently.

(a) [3 pts.] If $x[n] = (1/2)^n \cos(\pi n) u[n]$ find the energy E_∞ and the power P_∞.

(b) [3 pts.] Find the Z-transform, including the region of convergence (ROC), of the signal $x[n] = 3^n u[-n+2]$. Do this by a direct calculation from the series defining the Z-transform, e.g., do *not* use the transform table or properties directly.

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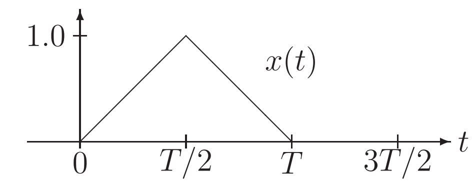
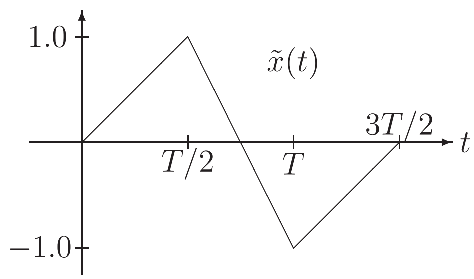
(c) [3 pts.] A continuous-time, linear and time-invariant (LTI) system has output $y(t) = e^{-at}\sin(\omega_0 t)u(t)$ when the input $x(t)$ is the triangular waveform shown. Find the output $\tilde{y}(t)$ when the input is $\tilde{x}(t)$. (There is *no* need to spend time trying to simplify the final form of $\tilde{y}(t)$.)

input **output**



$y(t) = e^{-at}\sin(\omega_0 t)u(t)$



$\tilde{y}(t) = ?$

(d) [3 pts.] Let $x(t)$ be periodic with fundamental period T. Suppose the Fourier series coefficients of $x(t)$ are denoted by x_n. Define a new periodic function by $\bar{x}(t) = x(t - T/2)$ and let its Fourier series coefficients be denoted by $\bar{x}_n$. Derive the relationship between $\bar{x}_n$ and x_n.

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(e) [3 pts.] An LTI system has an impulse response $h[n] = (1/3)^n u[n]$. Find the impulse response of an inverse system using Z transform techniques. Is the inverse system unique?

(f) [3 pts.] Calculate the discrete-time Fourier transform (DTFT) of $f[n] = e^{-10n}\cos(5n)u[n]$.

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Problem 2. [15 pts. total] True or false. For each statement below, indicate if it is true or false, explain your answer, including any conditions that must be added to make the statement precise (if needed).

(a) [3 pts.] T or F: If $f(t) \leftrightarrow F(s)$ is a time signal - bilateral Laplace transform pair, then the continuous time Fourier transform of $f(t)$ is always given by

$$F(s)|_{s=j\omega}\,.$$

(b) [3 pts.] T or F: Let $g(t)$ be a periodic signal with Fourier series coefficients G_n. Then

$$|G_n| \to 0, \quad \text{as } n \to \infty.$$

(c) A system with input $x(t)$ and output $y(t)$ is defined by

$$y(t) = \int_{-\infty}^{\infty} (t-\tau)\tau^2 u(t-\tau)x(\tau)\mathrm{d}\tau$$

where $u(\cdot)$ is the unit step.

(c-i) [1 pt.] T or F: The system is causal.

(c-ii) [1 pt.] T or F: The system is time-invariant.

(c-iii) [1 pt.] T or F: The system is linear.

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(d) [3 pts.] T or F: The cascade (i.e., series) connection of two nonlinear systems is itself nonlinear.

(e) [3 pts.] T or F: A Laplace transform $H(s)$ having N poles at $s = -1, -2, \ldots, -N$ has 2^{N+1} possible inverse transforms $h(t)$.

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Problem 3. [20 pts. total] A discrete-time filter has a transfer function

$$H(z) = \frac{\alpha(1 - \beta z^{-1})}{1 - (1/2)z^{-1}}, \quad \text{with ROC: } |z| > 1/2.$$

(a) [15 pts.] Choose the constants α and β so that the following two conditions are met:

1. The steady-state filter output is $10x_1[k]$ when the input $x_1[k]$ is given by $x_1[k] = (-1)^k u[k]$.
2. The steady-state filter output is 0 when the input $x_2[k]$ is given by $x_2[k] = u[k]$.

(b) [5 pts.] Draw a block diagram realization of the digital filter using delay elements, adders, and constant multipliers. (Leave α and β general so the answer to this part is independent of the previous part.)

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Problem 4. [25 pts. total] The bilateral Laplace transform of a signal $y(t)$ has the form

$$Y(s) = \frac{2s}{s^3 + s^2 - 4s - 4}.$$

(a) [4 pts.] Find the roots of the denominator polynomial of $Y(s)$ and write the denominator in factored form.

(b) [1 pt.] Double check that your roots are correct by multiplying the factored form back out.

(c) [4 pts.] Find the partial fraction expansion of $Y(s)$.

(d) [1 pt.] How many possible inverse transforms $y(t)$ can correspond to $Y(s)$?

(e) [7 pts.] Find $y(t)$ assuming it is an absolutely integrable signal. Draw the ROC for $Y(s)$ corresponding to this choice of $y(t)$. Show all poles and zeros of $Y(s)$ on this drawing.

(f) Now suppose that the $y(t)$ from part (e) is the output of an LTI system with input $x(t) = \delta(t) - 2e^{-2t}u(t)$.

  (f-i) [3 pts.] Find the Laplace transform $X(s)$ of $x(t)$ including the ROC.

  (f-ii) [3 pts.] Find the system function $H(s)$, including its ROC, of the system that takes the input $x(t)$ and produces the output $y(t)$ of part (e).

(g) [2 pts.] If we now consider the input $x(t)$ from (f) to be fixed and vary the choice of ROC used with the system function $H(s)$ can we generate all of the possible inverse transforms $y(t)$ that were mentioned in part (d)? If your answer is yes, explain why. If your answer is no, indicate which inverse transform cannot be obtained in this way.

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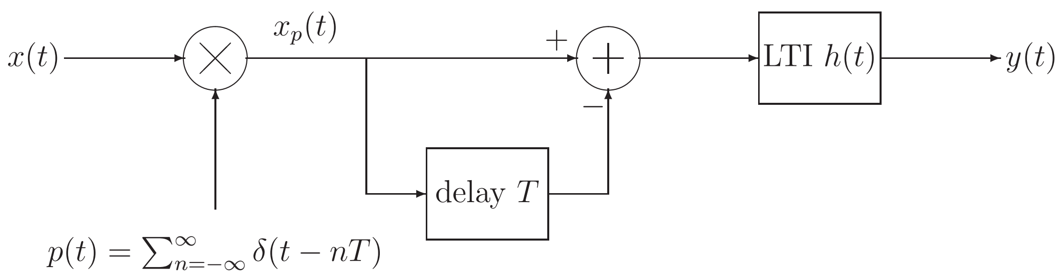
Problem 5. [30 pts. total] Consider the model of a sampler shown below, which is constructed from an ideal impulse sampler, a delay of T seconds, a summer, and an LTI filter with impulse response $h(t)$. The input signal to be sampled is $x(t)$ and the output signal is $y(t)$. Suppose we have the following continuous-time Fourier transform pairs: $x(t) \leftrightarrow X(j\omega)$, $x_p(t) \leftrightarrow X_p(j\omega)$, $h(t) \leftrightarrow H(j\omega)$, and $y(t) \leftrightarrow Y(j\omega)$.



(a) [3 pts.] Show that $y(t)$ can be written in the form

$$y(t) = \sum_{n=-\infty}^{\infty} x(nT)\tilde{h}(t - nT).$$

Give an expression relating $\tilde{h}(t)$ to $h(t)$.

(b) Derive the formulas for the following Fourier transforms:

(b-i) [3 pts.] $X_p(j\omega)$ in terms of $X(j\omega)$.

(b-ii) [2 pts.] $\tilde{H}(j\omega)$ in terms of $H(j\omega)$.

(b-iii) [2 pts.] $Y(j\omega)$ in terms of $X(j\omega)$ and $H(j\omega)$.

(c) Consider the case where $h(t) = u(t)$:

(c-i) [2 pts.] Show that the above sampler model is equivalent to what the O&W text called zero-order hold sampling.

(c-ii) [3 pts.] Find the transform $\tilde{H}(j\omega)$.

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(d) A perfect zero order hold is very difficult to build. As an approximation we may use $h(t) = e^{-at}u(t)$ where the time constant $1/a$ is large compared to T.

(d-i) [3 pts.] Assuming that $aT = 0.1$ draw a time-domain picture of $\tilde{h}(t)$ and use it to explain how this approximates the zero-order hold. Note that $e^{-0.1} \approx 0.9$.

(d-ii) [3 pts.] Find the transform $\tilde{H}(j\omega)$.

(e) [5 pts.] On the same axes draw the magnitude spectra of $\tilde{H}(j\omega)$ for the cases of parts (c) and (d). Compare and note the most significant difference between them.

(f) Consider processing the sampled signal $y(t)$ with a filter $H_r(j\omega)$ (not shown above) for the purpose of reconstructing $x(t)$. State the conditions on the input spectrum $X(j\omega)$ such that perfect reconstruction is theoretically possible:

(f-i) [2 pts.] In the case of zero-order hold sampling as in part (c).

(f-ii) [2 pts.] In the case of the approximation to the zero-order hold, which was defined in part (d).

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