

X. Summary of Some Basic Convolution Results

Additional Notes

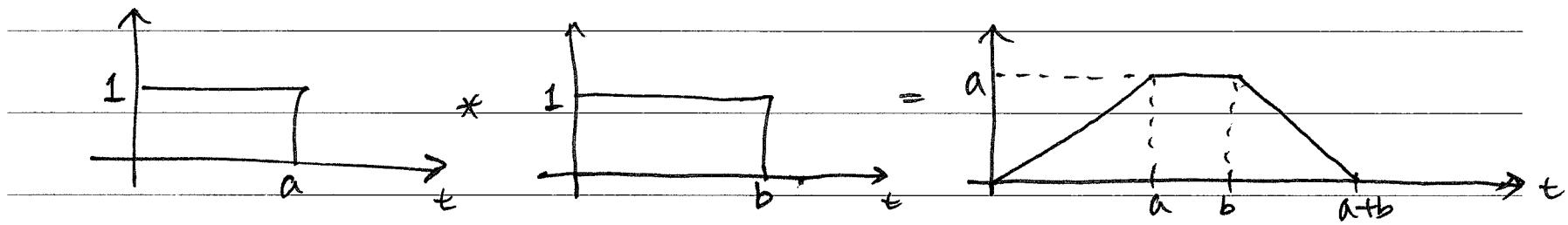
①

- $e^{at} u(t) * e^{bt} u(t)$

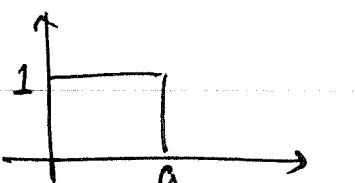
$$= \frac{1}{a-b} \{ e^{at} - e^{bt} \} u(t), \quad a, b: \text{complex-valued or real-valued and } a \neq b.$$

- $\text{rect}\left(\frac{t-\frac{a}{2}}{a}\right) * \text{rect}\left(\frac{t-\frac{b}{2}}{b}\right)$

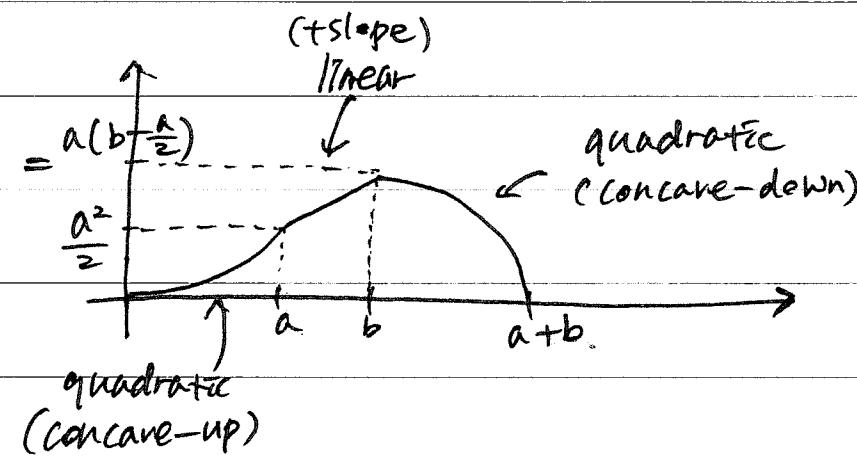
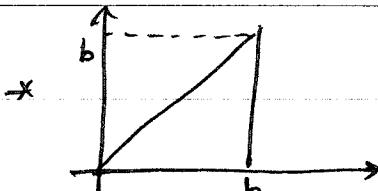
$$= t \text{rect}\left(\frac{t-\frac{a}{2}}{a}\right) + a \text{rect}\left(\frac{t-\frac{a+b}{2}}{b-a}\right) + (b+a-t) \text{rect}\left(\frac{t-(b+\frac{a}{2})}{2}\right)$$



- "Ramp-Up Triangle"



$(a < b)$

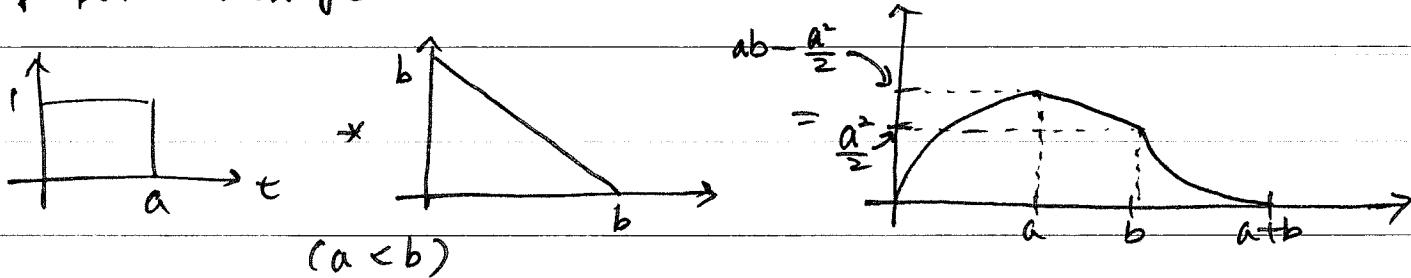


$$\{u(t) - u(t-a)\} * t \{u(t) - u(t-b)\}$$

$$= \frac{t^2}{2} \{u(t) - u(t-a)\} + (at - \frac{a^2}{2}) \{u(t-a) - u(t-b)\}$$

$$+ \left(-\frac{t^2}{2} + at + \frac{b^2 - a^2}{2} \right) \{u(t-b) - u(t-(a+b))\}$$

◦ "Ramp-down Triangle"



$$\{u(t) - u(t-a)\} * [-(t-b)\{u(t) - u(t-b)\}]$$

$$= \left(-\frac{t^2}{2} + bt \right) (u(t) - u(t-a)) + \left(-at + \frac{a^2}{2} + ab \right) \{u(t-a) - u(t-b)\}$$

$$+ \left\{ \frac{t^2}{2} - (a+b)t + \frac{(a+b)^2}{2} \right\} \{u(t-b) - u(t-(a+b))\}$$

◦ If $y(t) = x(t) * h(t)$

$$\text{then } a x(t-t_1) * b x(t-t_2) = ab y(t-(t_1+t_2))$$

• If $y_i(t) = x_i(t) * h(t)$, $i=1,2$

then $\{ax_1(t-t_1) + bx_2(t-t_2)\} * h(t)$
 $= ay_1(t-t_1) + by_2(t-t_2)$

then $\{ax_1(t-t_1) + bx_2(t-t_2)\} * h(t-t_0)$
 $= ay_1(t-(t_1+t_0)) + by_2(t-(t_2+t_0))$

* Summary of some Basic Properties of Convolution

1. Commutative : $x(t) * h(t) = h(t) * x(t)$

2. Associative : $(x(t) * h_1(t)) * h_2(t)$

$$= x(t) * (h_1(t) * h_2(t)) = x(t) * (h_2(t) * h_1(t))$$

Two LTI systems in series :

1) order doesn't matter

2) can be replaced by a single LTI system

$$h(t) = h_1(t) * h_2(t)$$

3. Distributive : $x(t) * (h_1(t) + h_2(t))$

$$= x(t) * h_1(t) + x(t) * h_2(t)$$

Two LTI systems in parallel : can be replaced by a
single system $h(t) = h_1(t) + h_2(t)$
