5. Let $G_n = (1/n, 1)$ $\Rightarrow$ G_n open.

$\forall x \in (0,1)$ $\quad 0 < x \Rightarrow \exists n \in \mathbb{N} \ni \frac{1}{n} < x \Rightarrow x \in G_n$

$\Rightarrow \bigcup_{n=1}^{\infty} G_n \supset (0,1)$, so $\{G_n\}$ is an open cover.

Let $G_{n_1}, \ldots, G_{n_k}$ be any finite sub-collection.

$\Rightarrow \bigcup_{j=1}^{k} G_{n_j} = (1/N, 1)$ where $N = \max_{j=1,\ldots,k} \{n_j\}$

$\Rightarrow \{G_n\}$ has no finite sub cover $\boxtimes$

6. Let $\{a_n\}, \{b_n\}$ sequences in $\mathbb{R}$, $a_n \to L$ & $|a_n - b_n| < 1/n$.

Let $\varepsilon > 0$ be given. $\exists N \ni \forall n \geq N$ $|a_n - L| < \varepsilon/2$

Choosing N larger we may assume $1/N < \varepsilon/2$.

$\Rightarrow |b_n - L| \leq |b_n - a_n| + |a_n - L| < 1/n + \varepsilon/2 < \varepsilon \quad \forall n \geq N.$

$\boxtimes$

7. (a) A compact & B closed.

$\Rightarrow$ A closed. $\Rightarrow$ $A \cap B$ closed subset of A, compact.

$\Rightarrow$ $A \cap B$ compact ✓ (True)

(b) Let $A = (0,1)$ $B = \mathbb{R}$. $\Rightarrow A \cap B = A$ not closed (False).

(c) A & B open, $B \subset A$

$A = (0,1) = B$ $\Rightarrow \overline{B} \not\subset A$ (False)

(d) A open, B closed, $B \subset A$ $\Rightarrow B' \subset \overline{B} = B \subset A$ ✓ (True)