Midterm Examination 1
ECE 301

Division 1, Spring 2007

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Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. You have 50 minutes to complete the 5 questions contained in this exam, for a total of up to 80 points. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
3. This booklet contains 9 pages. The last three pages contain a table of formulas and properties. You may tear out these three pages **once the exam begins**.
4. This is a closed book exam. The use of calculators is prohibited. Cell phones, pagers, and all other electronic communication device are strictly forbidden. Ipods and PDAs are not allowed either.

Name: ______________________________

Email: ______________________________

Signature: ______________________________

Itemized Scores

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Total:

(15 pts) **1.** Compute the energy E_∞ and the power P_∞ for the CT signal

$$x(t) = e^{j(2t+\frac{\pi}{4})}.$$

(Justify your answer.)

(15 pts) **2.** An LTI system has unit impulse response $h[n] = 3u[n]$. Compute the system's response to the input $x[n] = \left(\frac{1}{2}\right)^{n-1} u[n]$. (Justify your answer.)

(20 pts) **3.** Consider the system whose input $x(t)$ is related to the output $y(t)$ by the equation

$$y(t) = x(t-3) + x(3-t).$$

a) Check all properties that that hold for this system. (No justification needed.)

memoryless	
linear	
causal	
stable	

b) Is the above system time invariant? Answer yes/no and justify your answer.

(15 pts) **4.** Suppose we are given the following information about a signal $x[n]$:

1. $x[n]$ is a real and even signal.
2. x[n] has period $N = 10$ and Fourier coefficients a_k.
3. $a_{11} = 5$.
4. $\frac{1}{10} \sum_{n=0}^{9} |x[n]|^2 = 50$.

Find $x[n]$. (Justify your answer.)

(15 pts) **5.** Consider a continuous-time LTI system whose frequency response is

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} de = \frac{sin(4\omega)}{\omega}.$$

If the input to this system is a periodic signal

$$x(t) = \begin{cases} 1, & 0 \leq t < 4 \\ -1, & 4 \leq t < 8 \end{cases}$$

with period $T = 8$, determine the corresponding system output $y(t)$.

Facts and Formulas

1 CT Signal Energy and Power

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \tag{1}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \tag{2}$$

2 Fourier Series of CT Periodic Signals with period T

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\left(\frac{2\pi}{T}\right)t} dt \tag{4}$$

3 Properties of CT Fourier Series

Let $x(t)$ be a periodic signal with fundamental period T and fundamental frequency ω_0. Let $y(t)$ be another periodic signal with the same fundamental period T and fundamental frequency ω_0. Denote by a_k and b_k the Fourier series cofficients of x(t) and y(t) respectively.

	Signal	*FT*	
Linearity:	$\alpha x(t) + \beta y(t)$	$\alpha a_k + \beta b_k$	(5)
Time Shifting:	$x(t - t_0)$	$e^{-jk\omega_0 t_0} a_k$	(6)
Conjugation:	$x^*(t)$	a^*_{-k}	(7)
	$x(t)$ real and even	a_k real and even	(8)
	$x(t)$ real and odd	a_k pure imaginary and odd	(9)

Parseval's Relation: $$\frac{1}{T} \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \sum_{k=-\infty}^{\infty} |a_k|^2 \tag{10}$$

4 DT Signal Energy and Power

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 \tag{11}$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \tag{12}$$

5 Fourier Series of DT Periodic Signals with period N

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk(\frac{2\pi}{N})n} \tag{13}$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk(\frac{2\pi}{N})n} \tag{14}$$

6 Properties of DT Fourier Series

Let $x[n]$ be a periodic signal with fundamental period N and fundamental frequency ω_0. Let $y[n]$ be another periodic signal with the same fundamental period N and fundamental frequency ω_0. Denote by a_k and b_k the Fourier series cofficients of x(t) and y(t) respectively.

	Signal	*FT*	
Linearity:	$\alpha x[n] + \beta y[n]$	$\alpha a_k + \beta b_k$	(15)
Time Shifting:	$x[n - n_0]$	$e^{-jk\omega_0 n_0} a_k$	(16)
Conjugation:	$x^*[n]$	a^*_{-k}	(17)
	$x[n]$ real and even	a_k real and even	(18)
	$x[n]$ real and odd	a_k pure imaginary and odd	(19)
Parseval's Relation:	$\frac{1}{N} \sum_{n=0}^{N-1} \|x(t)\|^2 \, dt = \sum_{k=0}^{N-1} \|a_k\|^2$		(20)

7 Properties of LTI systems

- LTI systems commute.
- The response of an LTI system with unit impulse response h to a signal x is the same as the response of an LTI system with unit impulse response x to the signal h.
- An LTI system consisting of a cascade of k LTI systems with unit impulse responses $h_1, h_2, \ldots, h_k$ respectively, is the same as an LTI system with unit impulse response $h_1 * h_2 * \ldots * h_k$.
- The response of a CT LTI system with unit impulse response $h(t)$ to the signal e^{st} is $H(s)e^{st}$ where $H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau} d\tau$.
- The response of a DT LTI system with unit impulse response $h[n]$ to the signal z^n is $H(z)z^n$ where $H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k}$.