Midterm Examination 1
ECE 301
Division 1, Spring 2007
Instructor: Mimi Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.

2. You have 50 minutes to complete the 5 questions contained in this exam, for a total of up to 80 points. **When the end of the exam is announced, you must stop writing immediately.** Anyone caught writing after the exam is over will get a grade of zero.

3. This booklet contains 9 pages. The last three pages contain a table of formulas and properties. You may tear out these three pages **once the exam begins.**

4. This is a closed book exam. The use of calculators is prohibited. Cell phones, pagers, and all other electronic communication device are strictly forbidden. Ipods and PDAs are not allowed either.

Name:________________________________________
Email:________________________________________
Signature:____________________________________

**Itemized Scores**

Problem 1:
Problem 2:
Problem 3:
Problem 4:
Problem 5:
Total:
(15 pts) 1. Compute the energy $E_\infty$ and the power $P_\infty$ for the CT signal

$$x(t) = e^{j(2t + \frac{\pi}{4})}.$$ 

(Justify your answer.)
(15 pts) 2. An LTI system has unit impulse response $h[n] = 3u[n]$. Compute the system’s response to the input $x[n] = \left(\frac{1}{2}\right)^{n-1} u[n]$. (Justify your answer.)
(20 pts) 3. Consider the system whose input $x(t)$ is related to the output $y(t)$ by the equation

$$y(t) = x(t - 3) + x(3 - t).$$

a) Check all properties that hold for this system. (No justification needed.)

<table>
<thead>
<tr>
<th>Property</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>memoryless</td>
<td></td>
</tr>
<tr>
<td>linear</td>
<td></td>
</tr>
<tr>
<td>causal</td>
<td></td>
</tr>
<tr>
<td>stable</td>
<td></td>
</tr>
</tbody>
</table>

b) Is the above system time invariant? Answer yes/no and justify your answer.
(15 pts) **4.** Suppose we are given the following information about a signal $x[n]$: 

1. $x[n]$ is a real and even signal.
2. $x[n]$ has period $N = 10$ and Fourier coefficients $a_k$.
3. $a_{11} = 5$.
4. $\frac{1}{10} \sum_{n=0}^{9} |x[n]|^2 = 50$.

Find $x[n]$. (Justify your answer.)
(15 pts) 5. Consider a continuous-time LTI system whose frequency response is

\[ H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t} dt = \frac{\sin(4\omega)}{\omega}. \]

If the input to this system is a periodic signal

\[ x(t) = \begin{cases} 
1, & 0 \leq t < 4 \\
-1, & 4 \leq t < 8 
\end{cases} \]

with period \( T = 8 \), determine the corresponding system output \( y(t) \).
Facts and Formulas

1  CT Signal Energy and Power

\[ E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \]  

(1)

\[ P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \]  

(2)

2  Fourier Series of CT Periodic Signals with period T

\[ x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j k (\frac{2\pi}{T})t} \]  

(3)

\[ a_k = \frac{1}{T} \int_{0}^{T} x(t) e^{-j k (\frac{2\pi}{T})t} \, dt \]  

(4)

3  Properties of CT Fourier Series

Let \( x(t) \) be a periodic signal with fundamental period \( T \) and fundamental frequency \( \omega_0 \). Let \( y(t) \) be another periodic signal with the same fundamental period \( T \) and fundamental frequency \( \omega_0 \). Denote by \( a_k \) and \( b_k \) the Fourier series coefficients of \( x(t) \) and \( y(t) \) respectively.

<table>
<thead>
<tr>
<th>Signal</th>
<th>FT</th>
</tr>
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<tbody>
<tr>
<td>Linearity: ( \alpha x(t) + \beta y(t) )</td>
<td>( \alpha a_k + \beta b_k )</td>
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<tr>
<td>Time Shifting: ( x(t - t_0) )</td>
<td>( e^{-j k \omega_0 t_0} a_k )</td>
</tr>
<tr>
<td>Conjugation: ( x^*(t) )</td>
<td>( a_k^* )</td>
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</tr>
<tr>
<td>( x(t) ) real and odd</td>
<td>( a_k ) pure imaginary and odd</td>
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Parseval’s Relation:  
\[ \frac{1}{T} \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \sum_{k=-\infty}^{\infty} |a_k|^2 \]  

(10)
4 DT Signal Energy and Power

\[ E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 \]  

\[ P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \]  

5 Fourier Series of DT Periodic Signals with period \( N \)

\[ x[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn} \]  

\[ a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} \]  

6 Properties of DT Fourier Series

Let \( x[n] \) be a periodic signal with fundamental period \( N \) and fundamental frequency \( \omega_0 \). Let \( y[n] \) be another periodic signal with the same fundamental period \( N \) and fundamental frequency \( \omega_0 \). Denote by \( a_k \) and \( b_k \) the Fourier series coefficients of \( x(t) \) and \( y(t) \) respectively.

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Parseval’s Relation:  
\[ \frac{1}{N} \sum_{n=0}^{N-1} |x(t)|^2 \, dt = \sum_{k=0}^{N-1} |a_k|^2 \]
Properties of LTI systems

- LTI systems commute.
- The response of an LTI system with unit impulse response \( h \) to a signal \( x \) is the same as the response of an LTI system with unit impulse response \( x \) to the signal \( h \).
- An LTI system consisting of a cascade of \( k \) LTI systems with unit impulse responses \( h_1, h_2, \ldots, h_k \) respectively, is the same as an LTI system with unit impulse response \( h_1 * h_2 * \ldots * h_k \).
- The response of a CT LTI system with unit impulse response \( h(t) \) to the signal \( e^{st} \) is \( H(s)e^{st} \) where \( H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau \).
- The response of a DT LTI system with unit impulse response \( h[n] \) to the signal \( z^n \) is \( H(z)z^n \) where \( H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k} \).