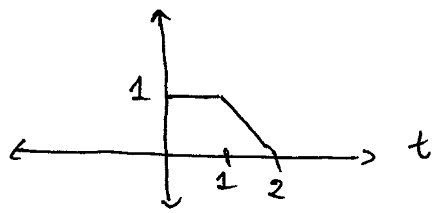
Problems

1.) $x(t) =$

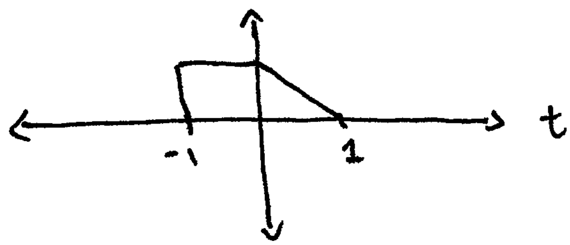


(a) $x\left(-\frac{t}{2}+1\right)$

(i) $x(t+1)$

$t_0 = -1 < 0$

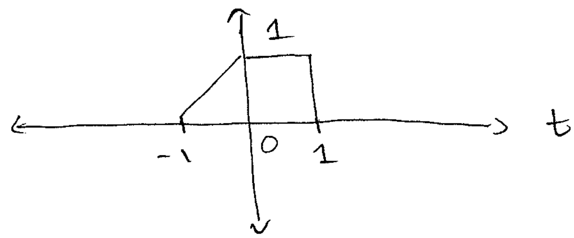
$\therefore$ Advance



(ii) $x(-t+1) =$

$x(1-t)$

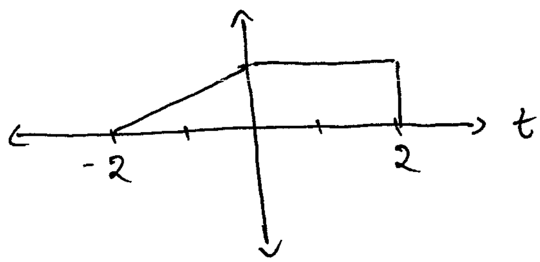
$\therefore$ Reversal



(iii) $x\left(-\frac{t}{2}+1\right)$

$\alpha < 1$

$\therefore$ Slower and signal will expand

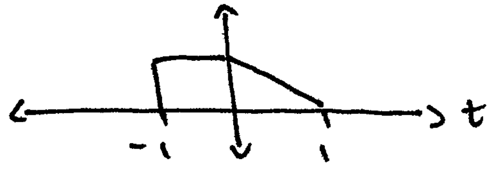


(b) $x\left(\frac{3}{2}t+1\right)$

(i) $x(t+1)$

$t_0 = -1 < 0$

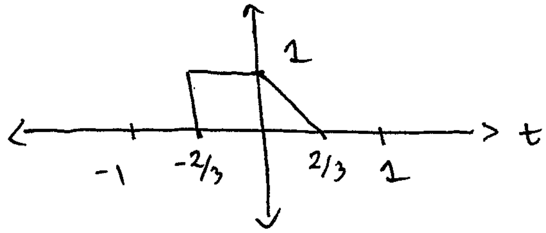
$\therefore$ Advance



(ii) $x\left(\frac{3}{2}t+1\right)$

$\alpha = \frac{3}{2} > 0$

$\therefore$ Faster and signal will shrink

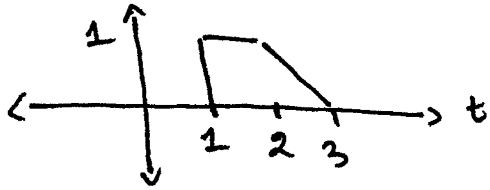


(c) $x(2t-1)$

(i) $x(t-1)$

$t_0 = 1 > 0$

$\therefore$ Delay

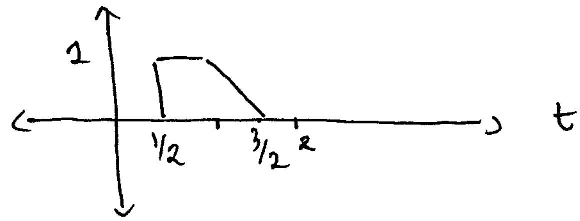


(ii) $x(2t-1)$

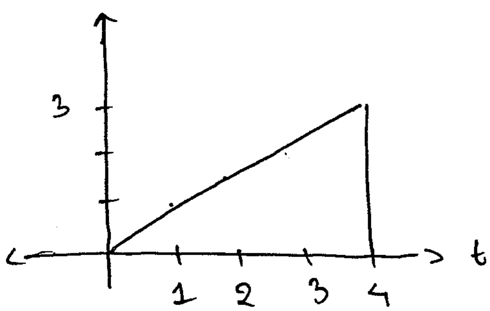
$\alpha = 2 > 1$

$\therefore$ Faster

Signal Shrinkness



2.)

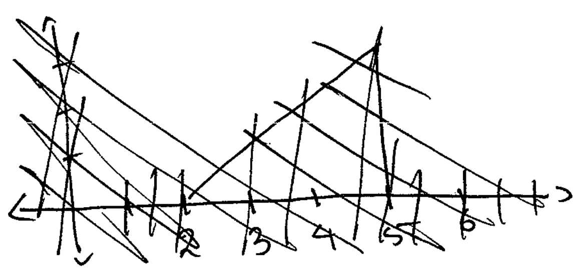


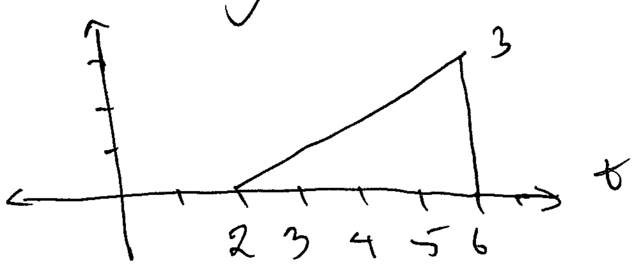
$= x(t)$

(a) $x(t-2)$

$t_0 = 2 > 0$

~~Advance~~ Delay



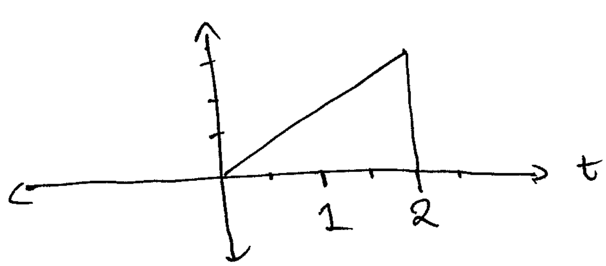


(b) $x(2t)$

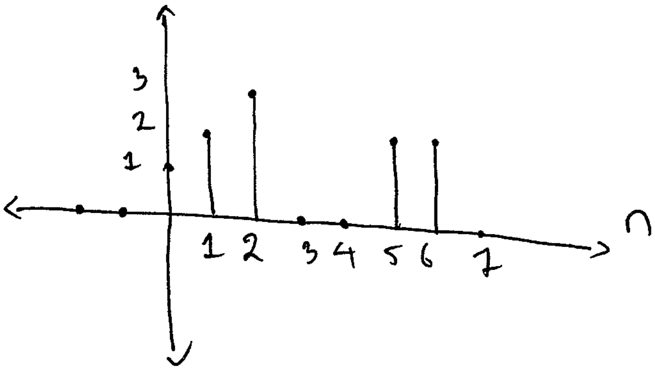
$\alpha = 2 > 0$

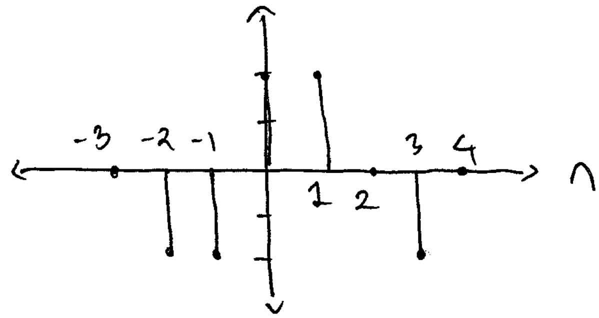
Increase

Signal w/ shrink

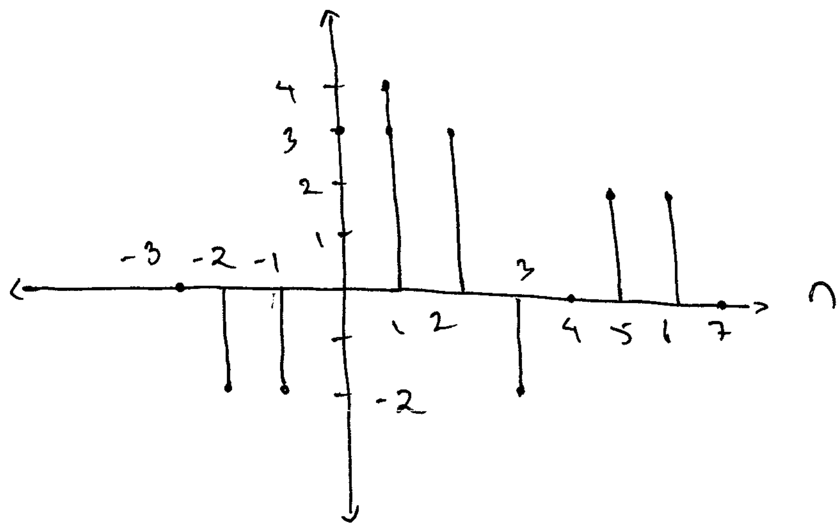


3.) $x_1[n]$ $x_2[n]$

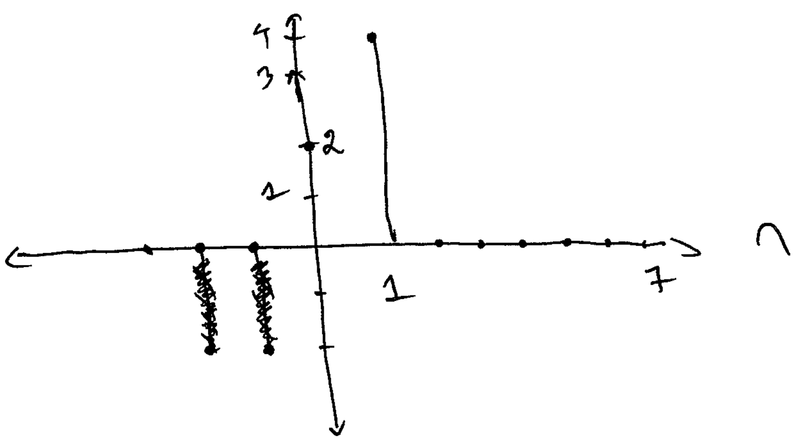




(a) $y_1[n] = x_1[n] + x_2[n]$



(b) $y_2[n] = x_1[n]\, x_2[n]$



4.) $x(t) = e^{jt}$

Find even & odd components

$$x(t) = \mathcal{E}v\{x(t)\} + \mathcal{O}d\{x(t)\}$$

$$= \frac{x(t) + x(-t)}{2} + \frac{x(t) - x(-t)}{2}$$

$$= \frac{e^{jt} + e^{-jt}}{2} + \frac{e^{jt} - e^{-jt}}{2}$$

$$= \cos t + j\sin t$$

5.) Show that

(a) $x(t)$ = even, then

$$\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$$

Proof- $$\int_{-a}^{a} x(t)dt = \int_{-a}^{0} x(t)\,dt + \int_{0}^{a} x(t)dt$$

(i) $$\int_{-a}^{0} x(t)\,dt = \int_{-a}^{0} x(-\tau)(-d\tau)$$

$$= \int_{0}^{a} x(-t)\,d\tau$$

because $x(t)$ is even signal

$$= \int_{0}^{a} x(\tau)d\tau$$

$$= \int_{0}^{a} x(t)dt$$

$$\therefore \int_{-a}^{a} x(t)\,dt = \int_{0}^{a} x(t)dt + \int_{0}^{a} x(t)dt$$

$$= 2\int_{0}^{a} x(t)\,dt$$

(b) $x[n]$ = even, then

$$\sum_{n=-k}^{k} x[n] = x[0] + 2\sum_{n=1}^{k} x[n]$$

$$\sum_{n=-k}^{k} x[n] = \sum_{n=-k}^{-1} x[n] + x[0] + \sum_{n=1}^{k} x[n]$$

$$\sum_{n=-k}^{-1} x[n] = \sum_{n=-k}^{-1} x[-n]$$

$$= \sum_{n=1}^{k} x[n]$$

$$\therefore \sum_{n=-k}^{k} x[n] = x[0] + 2\sum_{n=1}^{k} x[n].$$

6.) $x(0) = 0$; $x[0] = 0$; $x(t) = x[n] =$ Odd

(a) Prove $\rightarrow \int_{-a}^{a} x(t)\,dt = 0$

$$\int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$

$$\int_{-a}^{0} x(t)\,dt = \int_{-a}^{0} x(-t)(-dt)$$

$$= \int_{0}^{a} x(-t)\,dt$$

$x(t)$ is an odd signal

$$= \int_{0}^{a} -x(t)\,dt$$

$x(-t) = -x(t)$

$$= -\int_{0}^{a} x(t)\,dt$$

$$\therefore \int_{-a}^{a} x(t)\,dt = -\int_{0}^{a} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$

$$= 0. \quad \text{Ans}$$

(b)

(b) <u>Given</u>- $x[n]$ is an odd signal

$x[0] = 0$

<u>Prove</u>- $\sum_{n=-k}^{k} x[n] = 0$

<u>Proof</u>- $\sum_{n=-k}^{k} x[n] = \sum_{n=-k}^{-1} x[n] + x[0] + \sum_{n=1}^{k} x[n]$

$$\sum_{n=-k}^{-1} x[n] = \sum_{m=1}^{k} x[-m]$$

$$= \sum_{m=1}^{k} -x[+m]$$

$$= -\sum_{m=1}^{k} x[+m]$$

$$= -\sum_{n=1}^{k} x[n]$$

$$\therefore \sum_{n=-k}^{k} x[n] = -\sum_{n=1}^{k} x[n] + x[0] + \sum_{n=1}^{k} x[n]$$

$$= \underline{\underline{0}}$$ Ans.

7.) $x(t) = e^{j\omega_0 t}$ [Find the fund. period]

$x(t+T) = e^{j\omega_0 (t+T)}$

$= e^{j\omega_0 t} e^{j\omega_0 T}$

$= e^{j\omega_0 t}$

$$\left\{ \begin{array}{l} e^{j\omega_0 T} = 1 \\ \text{if} \end{array} \right.$$

1.) $\omega_0 = 0 \;, \rightarrow \; x(t) = 1$

2) $\omega_0 \neq 0 \;, \longrightarrow \; \omega_0 T = m 2\pi$

$$T = m \frac{2\pi}{\omega_0}.$$

8.) $x(t) = e^{j\omega_0 t}$

$$T_0 = \frac{2\pi}{\omega_0}$$

$\longrightarrow \; x[n] = x(nT_s) = e^{j\omega_0 (nT_s)}$

$$e^{j\omega_0 (n+N_0)T_s} = e^{j\omega_0 n T_s} e^{j\omega_0 N_0 T_s}$$

$$= e^{j\omega_0 n T_s}$$

$\therefore$ It is periodic

$\longrightarrow \; e^{j\omega_0 N_0 T_s} = 1$

$$\omega_0 N_0 T_s = m 2\pi$$

$$\frac{2\cancel{\pi}}{T_0} N_0 T_s = m 2\cancel{\pi}$$

$$\frac{m}{N_0} = \frac{T_s}{T_0} = \text{some rational \#}$$

9.) $x(t) = \cos 15t$

a.) $x[n] = x(nT_s)$. Find T_s

$$T_0 = \frac{2\pi}{\omega_0}$$

$$= \frac{2\pi}{15}$$

$$\frac{m}{N_0} = \frac{T_s}{T_0}$$

$$T_s = \frac{m}{N_0} T_0$$

$$= \frac{m}{N_0} \cdot \frac{2\pi}{15}$$

(b) $T_s = 0.1\pi$

$$T_s = \frac{m}{N_0} \frac{2\cancel{\pi}}{15} = 0.1\cancel{\pi}$$

$$\therefore \frac{m}{N_0} = \frac{0.1 \times 15}{2} = 3/4 = \cancel{0.75}$$

$$N_0 = m \times 4/3$$

$$= 3 \times 4/3 = \underline{\underline{4}}$$

10.) Determine periodicity & T_0

(a) $x(t) = \cos\left(t + \pi/4\right)$

$\omega_0 = 1$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{1} = 2\pi$$

(b) $x(t) = \sin\left(\frac{2\pi}{3}t\right)$

$\omega_0 = 2\pi/3$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi/3} = 3$

(c) $x(t) = \cos \pi/3\, t + \sin \pi/4\, t$

$\omega_0 = \pi/3$ $\qquad$ $\omega_0 = \pi/4$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\cancel{\pi}}{\cancel{\pi}/3}$ $\qquad$ $T_0 = 2\cancel{\pi}/\cancel{\pi}/4$

$= 6$ $\qquad$ $= 8$

$\therefore$ LCM $(6, 8) = 24$

(d) $x(t) = \cos t + \sin \sqrt{2}t$

$\omega_0 = 1$ $\qquad$ $\omega_0 = \sqrt{2}$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{1} = 2\pi$ $\qquad$ $T_0 = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$

$\therefore \frac{T_1}{T_2} = \frac{2\cancel{\pi}}{\sqrt{2}\cancel{\pi}} = \sqrt{2}$ (Irrational)

$\therefore$ Non-periodic

(e) $x(t) = \sin^2 t = (\sin t)^2 = \frac{1}{2}(1 - \cancel{2}\cos 2\theta)$

~~$\omega_0 = 1$~~ $\qquad = 1/2 - 1/2 \cos 2\theta$

~~$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{1} = 2\pi$~~

$\omega_0 = 2$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi.$ $\qquad$ $\therefore$ Periodic

(f) $e^{j[(\pi/2)t-1]} = \dfrac{e^{j\pi/2 t}}{e^{+j}} = e^{j\pi/2 t} e^{-j}$

$\omega_0 = \pi/2$

$T_0 = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{\pi/2} = 4$

(g) $x[n] = e^{j(\pi/4)n}$

$\omega_0 = \pi/4$

$K/N = \dfrac{\omega_0}{2\pi} = \dfrac{\pi/4}{2\pi} = 1/8 \times 8 = 1$

$\therefore N_0 = 8$

(h) $x[n] = \cos 1/4\, n$

$\omega_0 = 1/4$

$K/N = \dfrac{\omega_0}{2\pi} = \dfrac{1/4}{2\pi} = 1/8\pi$ (Irrational)

$\therefore$ Non-Periodic

(i) $x[n] = \cos \pi/3\, n + \sin \pi/4\, n$

$\omega_1 = \pi/3$

$K/N = \dfrac{\pi/3}{2\pi} = \dfrac{1}{6} \times 6$

$\omega_2 = \pi/4$

$K/N = \dfrac{\pi/4}{2\pi} = \dfrac{1}{8} \times 8$

$\therefore N_0 = LCM(6,8) = \underline{24}$

(j) $x[n] = \cos^2 \pi/8\, n = 1/2 + 1/2 \cos \pi/4\, n$

$N_1 = 1$

$\omega_0 = \pi/4$

$K/N = \dfrac{\pi/4}{2\pi} = 1/8 \times 8$

$\therefore N_0 = \underline{8}$

11.) (a) Prove $\int_{\alpha}^{\beta} x(t)\,dt = \int_{\alpha+T}^{\beta+T} x(t)\,dt$

$t = \alpha$

$t = \alpha + T$

$\alpha = t - T$

$$= \int_{\alpha+T}^{\beta+T} x(t-T)\,dt \rightarrow (1)$$

$x(t) = x(t+T)$ { given }

Let $t = \tau - T$

$= x(\tau - T + T)$

$= x(\tau)$

$= x(t)$

$= x(\tau - T) \rightarrow (2)$

Plugging (2) in (1);

$$\int_{\alpha}^{\beta} x(t)\,dt = \int_{\alpha+T}^{\beta+T} x(t)\,dt$$

$\therefore$ Thus, proved. Ans.

(b) $\int_{a}^{a+T} x(t) = \int_{0}^{T} x(t)$

$$\int_{a}^{a+T} x(t)\,dt = \int_{a}^{0} x(t)\,dt + \int_{0}^{a+T} x(t)\,dt$$

$t = a$

If $t = a + T$

$t - T = a$

$$= \int_{a+T}^{T} x(\tau - T)\, dt + \int_{0}^{a+T} x(t)\, dt$$

$$= \int_{0}^{a+T} x(t)\, dt + \int_{a+T}^{T} x(t)\, dt$$

$$= \int_{0}^{T} x(t)\, dt$$

$\therefore$ Thus, proved.

Ans.

$$x(t) = x(t+T)$$

$$t = \tau - T$$

$$x(t) = x(\tau - T + T)$$

$$= x(\tau) = x(t)$$

$$= x(\tau - T)$$

12.) Prove:-

(a) $$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \dfrac{1-\alpha^N}{1-\alpha} & \alpha \neq 1 \\ N & \alpha = 1 \end{cases}$$

$$S = \sum_{n=0}^{N-1} \alpha^n = 1 + \alpha + \alpha^2 + \ldots + \alpha^{N-1} \rightarrow (1)$$

$$\alpha S = \alpha \sum_{n=0}^{N-1} \alpha^n = \alpha + \alpha^2 + \alpha^3 + \ldots \alpha^N \rightarrow (2)$$

Sub. (2) from (1);

$$(1-\alpha) S = 1 - \alpha^N$$

$$S = \frac{1-\alpha^N}{1-\alpha} = \sum_{n=0}^{N-1} \alpha^n$$

(b) $\sum_{n=0}^{\infty} a^n = \frac{1}{1-\alpha} \qquad |\alpha| < 1$

$$\lim_{N\to\infty} \sum_{n=0}^{N-1} \alpha^n = \lim_{N\to\infty} \frac{1-\alpha^N}{1-\alpha} = \frac{1}{1-\alpha}$$

(c) $\sum_{n=k}^{\infty} \alpha^n = \frac{\alpha^k}{1-\alpha} \qquad |\alpha| < 1$

$$\sum_{n=k}^{\infty} \alpha^n = \alpha^k + \alpha^{k+1} + \alpha^{k+2} + \cdots$$

$$= \alpha^k \left[1 + \alpha + \alpha^2 + \alpha^3 + \cdots \right]$$

$$= \alpha^k \sum_{n=0}^{\infty} \alpha^n$$

$$= \underline{\frac{\alpha^k}{1-\alpha}} \qquad \text{Ans.}$$

(d) $\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2}$

$$\frac{d}{d\alpha} \sum_{n=0}^{\infty} \alpha^n = \frac{d}{d\alpha}\left(\frac{1}{1-\alpha}\right)$$

$$\sum_{n=0}^{\infty} n\alpha^{n-1} = \frac{1}{(1-\alpha)^2}$$

$$\frac{1}{\alpha} \sum_{n=0}^{\infty} n\alpha^n = \frac{1}{(1-\alpha)^2}$$

$$\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2} . \qquad \text{Ans.}$$

13.)

(a) $x(t) = e^{-at} u(t) \qquad a > 0$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_0^{\infty} |e^{-at}|^2 \, dt = \int_0^{\infty} e^{-2at} \, dt$$

$$= \frac{e^{-2at}}{-2a} \Big]_0^{\infty} = -\frac{1}{-2a} = \frac{1}{2a} < \infty$$

$\therefore$ Energy signal

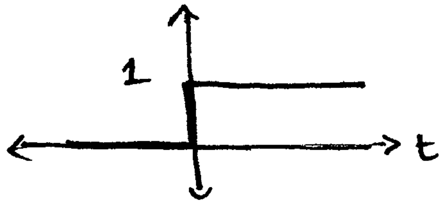
(b) $x(t) = A\cos(\omega_0 t + \theta)$

$$T_0 = \frac{2\pi}{\omega_0}$$

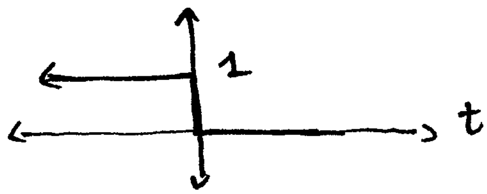
$$P = \frac{1}{T_0} \int_0^{T_0} |x(t)|^2 \, dt =$$

14.) $u(-t) = \begin{cases} 0 & t > 0 \\ 1 & t < 0 \end{cases}$

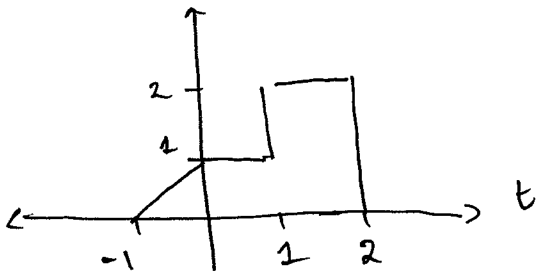
$u(t) =$



$u(-t) =$

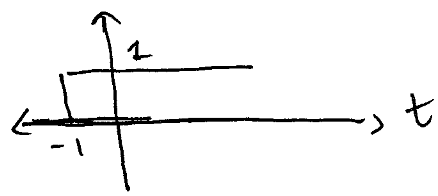


15.) $x(t) =$

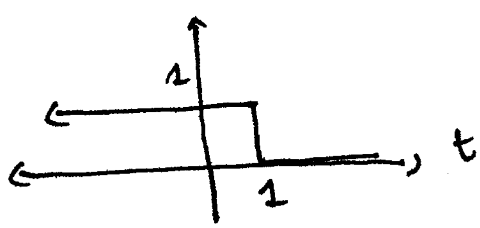


(a) $x(t)\,u(1-t)$

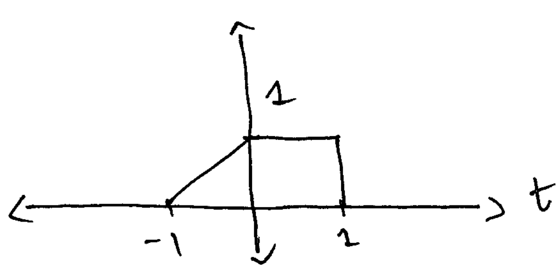
(i) $u(t+1)$



(ii) $u(-t+1)$

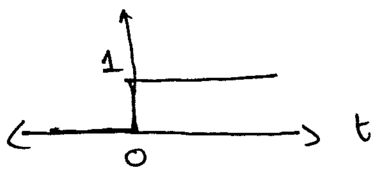


$\therefore$ (iii) $x(t)\,u(-t+1)$

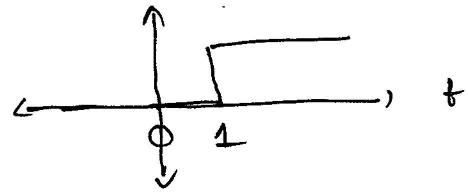


(b) $x(t)\left[u(t) - u(t-1)\right]$

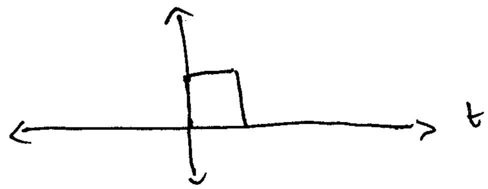
(i) $u(t)$



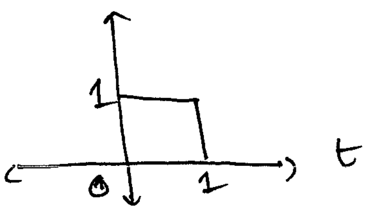
$u(t-1)$



(ii) $u(t) - u(t-1)$



(iii) $x(t)\left[u(t) - u(t-1)\right]$



16.) (a) $\int_{-1}^{1} (3t^2+1)\,\delta(t)\,dt \;=\; 3t^2+1 \Big]_{t=0} = 1$

(b) $\int_{1}^{2} (3t^2+1)\,\delta(t)\,dt = 0$

(c) $\int_{-2}^{-1} (3t^2+1)\,\delta(t)\,dt = 0$

(d) $\int_{-\infty}^{\infty} (t^2 + \cos \pi t)\,\delta(t-1)\,dt$

$= t^2 + \cos \pi t \,\Big|_{t=1}$

$= 1 + \cos \pi = 1 - 1 = \underline{\underline{0}}$

16. e.) $\int_{-\infty}^{\infty} e^{-t}\,\delta(2t-2)\,dt$

$2t-2=0$
$2t=2$
$t=1$

~~$= e^{-t}\Big|_{t=1} = e^{-t}$~~

$= \int_{-\infty}^{\infty} e^{-t}\,\cancel{2}\,\delta[2(t-1)]\,dt = \frac{1}{2}e^{-t}\Big|_{t=1} = \frac{1}{2e}$

(f) $\int_{-\infty}^{\infty} e^{-t}\,\delta'(t)\,dt = -\,d/dt\,\cancel{\delta(t)}\,(e^{-t})\Big]_{t=0}$

$= +\frac{e^{-t}}{+1} = e^{-t}\Big]_{t=0} = \underline{1}$

17.) $y = x^2$ { Show its non-linear }

$x_1(t) \xrightarrow{S} y_1(t) = x_1^2$

$x_2(t) \xrightarrow{S} y_2(t) = [x_2(t)]^2$

$x_3(t) = ax_1(t) + bx_2(t) \longrightarrow y_3(t) = [x_3(t)]^2$

$= [x_1(t) + x_2(t)]^2$

$= x_1(t)^2 + 2x_1(t)x_2(t) + x_2(t)^2$

$\therefore\ x_3(t) \neq y_3(t).$

18.) $y[n] = x[n-1]$

(a) Memoryless - No

(b) Causal - Causal

(c) $x_1[n] \xrightarrow{S} y_1[n] = x_1[n-1]$

$x_2[n] \xrightarrow{S} y_2[n] = x_2[n-1]$

linear $x_3[n] = x_1[n] + x_2[n] \xrightarrow{S} y_3[n] = x_3[n-1]$

$= x_1[n-1] + x_2[n-1]$

$= y_1[n] + y_2[n]$

(d) $x_1[n] \xrightarrow{s} y_1[n] = x_1[n-1]$

$$x_2[n] = x_1[n-N] \xrightarrow{s} y_2[n] = x_2[n-1] \rightarrow (1)$$

$$y_2[n] = y_1[n-N] = x_1[n-N-1] \rightarrow (2)$$

$$x_2[n-1] = x_1[n-1-N] \rightarrow (3)$$

$\therefore$ from (2) & (3)

$$y_2[n] = y_1[n-N].$$

(e) Bounded

19.) $y[n] = x[n-1] \cancel{+} y[n-1]$

$$y[n] - y[n-1] = x[n-1]$$

20.) $y[n] = nx[n]$

(a) Memoryless - yes

(b) Causal - yes

(c) $x_1[n] \xrightarrow{s} y_1[n] = n\,x_1[n]$

$$x_2[n] \xrightarrow{s} y_2[n] = nx_2[n]$$

$$x_3[n] = ax_1[n] + bx_2[n] \xrightarrow{s} y_3[n] = nx_3[n]$$

$$= n\{ax_1[n] + bx_2[n]\}$$

$$= nax_1[n] + bnx_2[n]$$

$$= ay_1[n] + by_2[n]$$

$\therefore$ Linear

(d) $x_1[n] \xrightarrow{s} y_1[n] = n x_1[n]$

$x_2[n] = x_1[n-N] \xrightarrow{s} y_2[n] = n x_2[n]$

$= n x_2[n-N] \rightarrow (1)$

$y_1[n-N] = y_2[n] = (n-N)\, x_1[n-N]$

$\therefore\ y_2[n] \neq y_1[n-N]$

(e) Bounded - No

21.) Prove linearity -

(i) $y(t) = x(t) + 1$

$x_1(t) \xrightarrow{s} y_1(t) = x_1(t) + 1$

$x_2(t) \xrightarrow{s} y_2(t) = x_2(t) + 1$

$x_3(t) = a x_1(t) + b x_2(t) \xrightarrow{s} y_3(t) = x_3(t) + 1$

$= a x_1(t) + b x_2(t) + 1 \rightarrow (1)$

$y_3(t) = a y_1(t) + b y_2(t)$

$= a[x_1(t) + 1] + b[x_2(t) + 1]$

$= a x_1(t) + a + b x_2(t) + b \rightarrow (2)$

$(1) \neq (2)$

$\therefore$ Not Linear

(ii) $y(t) = t x(t)$

$x_1(t) \xrightarrow{s} t x_1(t) = y_1(t)$

$x_2(t) \xrightarrow{s} t x_2(t) = y_2(t)$

$x_3(t) = a x_1(t) + b x_2(t) \xrightarrow{s} y_3(t) = t x_3(t) =$

$t a x_1(t) + t b x_2(t) =$

$a y_1(t) + b y_2(t)$

$\therefore$ Linear

22.) Prove 'Time Invariance' -

(a) $y(t) = t x(t)$

$x_1(t) \xrightarrow{s} y_1(t) = t x_1(t)$

$x_2(t) = x_1(t-t_0) \xrightarrow{s} y_2(t) = y_1(t-t_0)$

$= (t-t_0) x_1(t-t_0) \to (1)$

$y_2(t) = t x_2(t) = t x_1(t-t_0) \to (2)$

$(1) \neq (2)$

$\therefore$ Not Time Invariance

(b) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$x_1(t) \xrightarrow{s} y_1(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau$

$x_2(t) = x_1(t-t_0) \xrightarrow{s} y_2(t) = y_1(t-t_0)$

$= \int_{-\infty}^{t-t_0} \cancel{x(t+t_0)}\, d\tau\, x(\tau)$

$y_2(t) = \int_{-\infty}^{t} x_2(\tau)\, d\tau$

$= \int_{-\infty}^{t} x_1(\tau - t_0)\, d\tau$

If $\tau' = \tau - t_0$

$= \int_{\tau'=-\infty}^{t-t_0} x_1(\tau')\, d\tau' = \int_{\tau=-\infty}^{t-t_0} x(\tau)\, d\tau$

$\therefore$ It is Time Invariance

23.) $$\int_0^{2\pi} t \sin t/2 \, \delta(\pi - t)\, dt = t \sin t/2 \Big|_{t=\pi}$$

$$= \pi \sin \pi/2 = \underline{\underline{\pi}}$$

24.) (a) $$y(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau)\, d\tau$$

$$x_1(t) \xrightarrow{S} y_1(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x_1(\tau)\, d\tau$$

$$x_2(t) \xrightarrow{S} y_2(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x_2(\tau)\, d\tau$$

$$x_3(t) = a x_1(t) + b x_2(t) \xrightarrow{S} y_3(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x_3(\tau)\, d\tau \rightarrow (1)$$

$$= \frac{1}{T} \int_{t-T/2}^{t+T/2} [a x_1(\tau) + b x_2(\tau)]\, d\tau$$

$\therefore$ linear

$$= \frac{1}{T} a \int_{t-T/2}^{t+T/2} \cancel{a} x_1(\tau)\, d\tau + \frac{1}{T} b \int_{t-T/2}^{t+T/2} x_2(\tau)\, d\tau$$

$$= a\, y_1(t) + b\, y_2(t)$$

?? (b) $$x_1(t) \xrightarrow{S} y_1(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x_1(\tau)\, d\tau$$

$$x_2(t) = x_1(t - t_0) \xrightarrow{S} y_2(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x_2(\tau)\, d\tau$$

$$= \frac{1}{T} \int_{t-T/2}^{t+T/2} x_1(\tau - t_0)\, d\tau$$

$t = \tau - t_0$