

Student Name: _____

Grader Name: _____

Instructions: This is an open notes, open book, collaborative quiz. No Internet allowed.

Question: Let X be a one-dimensional normal random variable with mean equal to one and standard deviation equal to two. Use the standard normal table displayed to compute the following probabilities

- a) Prob ($X < 1.08$)
- b) Prob ($X > 1.02$)
- c) Prob ($1.02 < X < 1.08$)
- d) Prob ($0.98 < X < 1.18$)

(Give the numerical values for each probability and write the intermediate steps you followed to get these values.)

Answer: $Y = \frac{X-1}{2}$ is a standard normal variable.

$$a) \text{Prob}(X < 1.08) = \text{Prob}\left(Y < \frac{1.08-1}{2}\right) = \text{Prob}(Y < 0.04) = \Phi(0.04) \approx 0.5160$$

$$b) \text{Prob}(X > 1.02) = \text{Prob}\left(Y > \frac{1.02-1}{2}\right) = \text{Prob}(Y > 0.01) = 1 - \text{Prob}(Y < 0.01) = 1 - \Phi(0.01) \\ \approx 1 - 0.5040 = 0.4960$$

$$c) \text{Prob}(1.02 < X < 1.08) = \text{Prob}(0.01 < Y < 0.04) = \Phi(0.04) - \Phi(0.01) \\ \approx 0.5160 - 0.5040 = 0.0120$$

$$d) \text{Prob}(0.98 < X < 1.18) = \text{Prob}\left(\frac{0.98-1}{2} < Y < \frac{1.18-1}{2}\right) = \text{Prob}(-0.01 < Y < 0.09) \\ = \Phi(0.09) - \Phi(-0.01) = \Phi(0.09) - (1 - \Phi(0.01)) \\ \approx 0.5359 - 1 + 0.5040 = 0.0399$$

Grade out of 20 first, then divide by 2

~~2 pts per correct numerical answer~~ → 8

4 pts for correctly placed ≈ (1 per Q)

4 pts for flow, clarity, signs (1 per Q)

a) 2 pts (1 just, 1 num)

b) 3 pts (2 just, 1 num)

c) 3 pts (2 just, 1 num)

d) 4 pts (3 just, 1 num)

Score: / 10