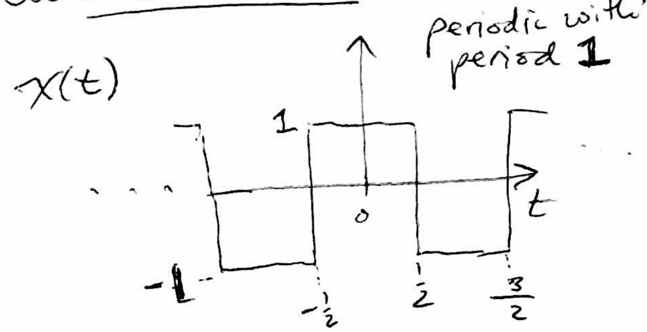
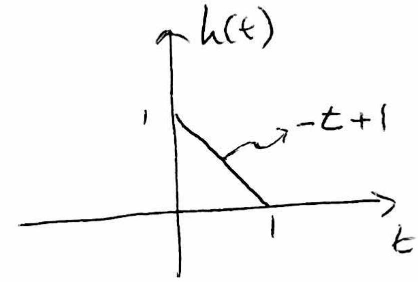
OW2.22 e





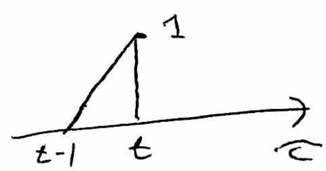
Find $x(t) * h(t)$.

We only need to look at a single period.

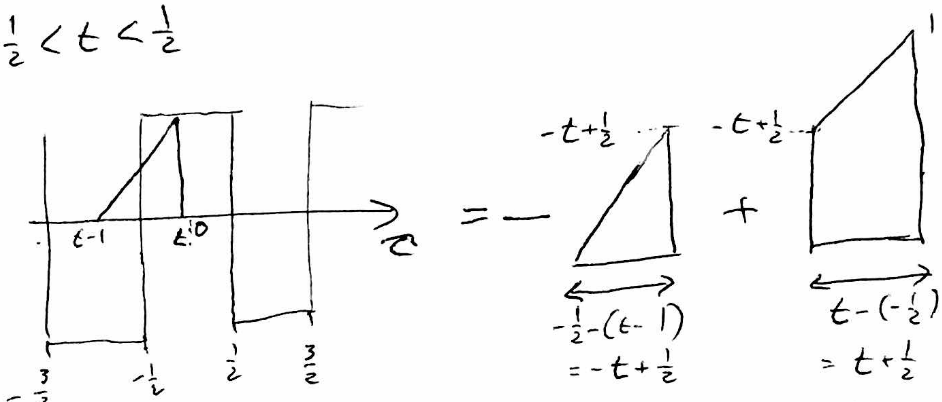
Choose $-\frac{1}{2} < t < \frac{3}{2}$

• Using the areas of the multiplication of $x(\tau)$ and $h(t-\tau)$:

$h(t-\tau)$



$-\frac{1}{2} < t < \frac{1}{2}$



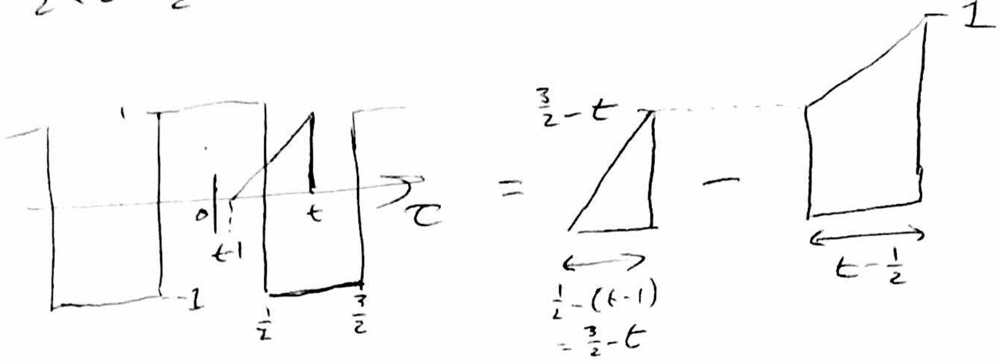
$$= -\underbrace{\frac{1}{2}\left(-t+\frac{1}{2}\right)^2}_{\text{area of triangle}} + \underbrace{\left(t+\frac{1}{2}\right)\left(\frac{1+\left(-t+\frac{1}{2}\right)}{2}\right)}_{\text{area of trapezoid}}$$

$$= -\frac{1}{2}\left(t^2 - t + \frac{1}{4}\right) + \left(t+\frac{1}{2}\right)\left(\frac{3}{4} - \frac{t}{2}\right)$$

$$= -\frac{t^2}{2} + \frac{t}{2} - \frac{1}{8} - \frac{t^2}{2} + \frac{3}{4}t - \frac{t}{4} + \frac{3}{8}$$

$$= -t^2 + t + \frac{1}{4}$$

$\frac{1}{2} < t < \frac{3}{2}$



$$= \frac{1}{2}\left(\frac{3}{2}-t\right)^2 - \left(t-\frac{1}{2}\right)\left(\frac{1+\frac{3}{2}-t}{2}\right)$$

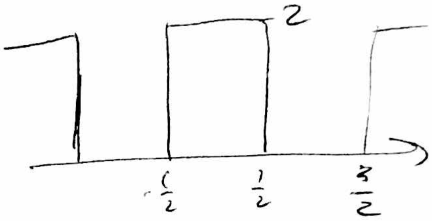
$$= \frac{1}{2}\left(\frac{9}{4}-3t+t^2\right) - \left(t-\frac{1}{2}\right)\left(\frac{5}{4}-\frac{t}{2}\right)$$

$$= \frac{9}{8} - \frac{3}{2}t + \frac{1}{2}t^2 - \left(-\frac{t^2}{2} + \frac{5}{4}t + \frac{1}{4}t - \frac{5}{8}\right)$$

$$= \frac{9}{8} - \frac{3}{2}t + \frac{1}{2}t^2 + \frac{t^2}{2} - \frac{5}{4}t - \frac{1}{4}t + \frac{5}{8}$$

$$= \frac{3}{2}t^2 - 3t + \frac{7}{4}$$

Alternative method:



from $x_1(t) = x(t) + 1$, then

$$x_1(t) * h(t) = x(t) * h(t) + 1 * h(t)$$
$$= x(t) * h(t) + \int_{-\infty}^{\infty} h(\tau)\,d\tau$$

and we have

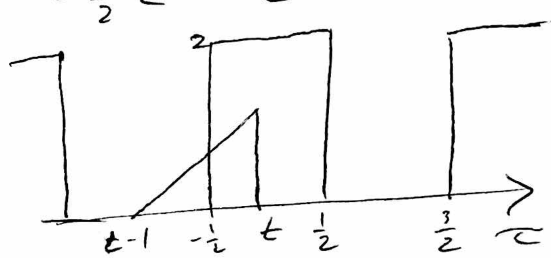
$$x(t) * h(t) = x_1(t) * h(t) - \int_{-\infty}^{\infty} h(\tau)\,d\tau$$

The desired convolution is $x_1(t) * h(t)$ minus a constant.

The constant is $\int_{-\infty}^{\infty} h(t)\,d\tau = \frac{1}{2}$ (the area of the ~~tr~~ triangle).

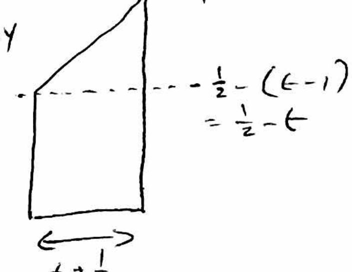
Then as previously:

$-\frac{1}{2} < t < \frac{1}{2}$



scaled by $x(\tau)$

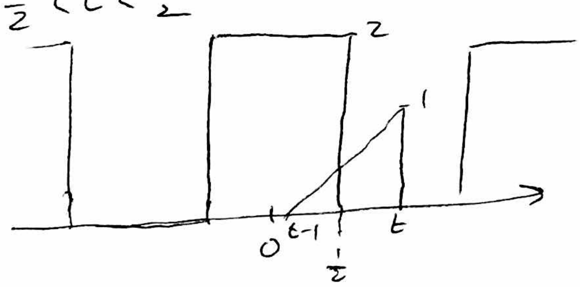
$= 2 \cdot$



$$= 2\left(t + \tfrac{1}{2}\right)\left(\frac{1 + \frac{1}{2} - t}{2}\right)$$
$$= 2\left(t + \tfrac{1}{2}\right)\left(\tfrac{3}{4} - \tfrac{t}{2}\right)$$
$$= 2\left(-\tfrac{t^2}{2} + \tfrac{3}{4}t - \tfrac{t}{4} + \tfrac{3}{8}\right)$$
$$= -t^2 + t + \tfrac{3}{4}$$

$$\Rightarrow x(t) * h(t) = \left(-t^2 + t + \tfrac{3}{4}\right) - \tfrac{1}{2} = -t^2 + t + \tfrac{1}{4} \quad \text{for } -\tfrac{1}{2} < t < \tfrac{1}{2}.$$

$\frac{1}{2} < t < \frac{3}{2}$



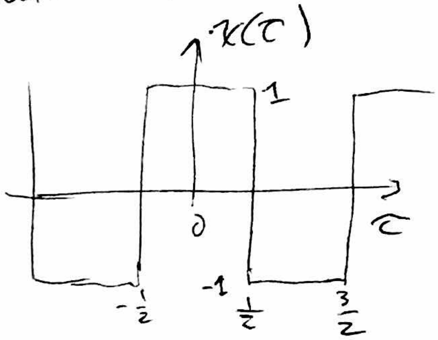
$= 2 \cdot$

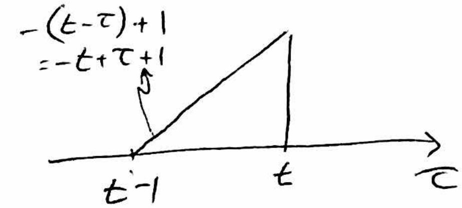
$\frac{1}{2} - (t-1)$
$= \frac{3}{2} - t$

$$= 2 \cdot \tfrac{1}{2}\left(\tfrac{3}{2} - t\right)^2 = \tfrac{9}{4} - 3t + t^2$$

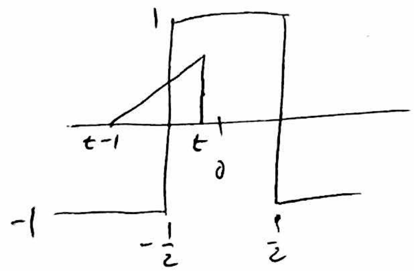
$$\Rightarrow x(t) * h(t) = \left(t^2 - 3t + \tfrac{9}{4}\right) - \tfrac{1}{2} = t^2 - 3t + \tfrac{7}{4}$$

Another method: setting up the integrals





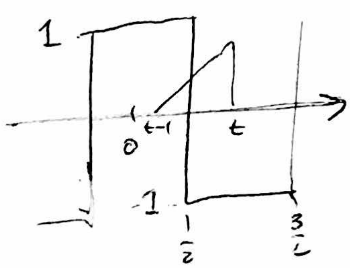
- $-\frac{1}{2} < t < \frac{1}{2}$



$$= \int_{t-1}^{-\frac{1}{2}} (-t+\tau+1)(-1)\,d\tau + \int_{-\frac{1}{2}}^{t} (-t+\tau+1)(1)\,d\tau$$

Not going to work all this out.

- $\frac{1}{2} < t < \frac{3}{2}$



$$= \int_{t-1}^{\frac{1}{2}} (-t+\tau+1)\,d\tau + \int_{\frac{1}{2}}^{t} (-t+\tau+1)(-1)\,d\tau$$