

ECE 302 Quiz 4  
(10 points)

Name: Solution (3 points)

07/15/2016

1. A student rolls two fair dice. Let the random variables  $X$  and  $Y$  denote the value of the first and second roll, respectively. Assume the rolls are independent. Show all work.

(a) (3 points) Find  $p_{X,Y}(x_i, y_j)$ . Indicate the values of  $x_i$  and  $y_j$ .

(b) (2 points) Find  $\mathbb{E}[XY]$ .

(c) (2 points) Find  $\mathbb{E}[X/Y]$ .

$$(a) P_X(x_i) = 1/6, \quad x_i = 1, 2, \dots, 6$$

$$P_Y(y_j) = 1/6, \quad y_j = 1, 2, \dots, 6$$

$X, Y$  independent

$$\Rightarrow P_{X,Y}(x_i, y_j) = P_X(x_i) P_Y(y_j)$$

$$= 1/36$$

$$x_i = 1, 2, \dots, 6$$

$$y_j = 1, 2, \dots, 6$$

$$(b) X, Y \text{ independent} \Rightarrow \mathbb{E}[XY] = \mathbb{E}[X] \mathbb{E}[Y]$$

$$= (7/2)^2$$

$$= \boxed{49/4}$$

$$(c) X, Y \text{ independent} \Rightarrow \mathbb{E}[X/Y] = \mathbb{E}[X] \mathbb{E}[1/Y]$$

$$\mathbb{E}[1/Y] = \frac{1}{6} \left( 1 + \frac{1}{2} + \dots + \frac{1}{6} \right)$$

$$= 49/20$$

$$\Rightarrow \mathbb{E}[X/Y] = 21 \cdot \boxed{7/2 \cdot 49/20}$$