1. Let $g \in C^1(\mathbb{R})$. Show g takes sets of measure zero to sets of measure zero.

2. If E_1, E_2 are measurable sets in $\mathbb{R}$, show $E_1 \times E_2$ is measurable in $\mathbb{R}^2$.

3. Let f be continuous on $[1, \infty)$ and $\lim_{x\to\infty} f(x) = 0$. Suppose that $\int_1^\infty f(x)x^n dx = 0$ for $n = -2, -3, \ldots$. Does it follow that $f \equiv 0$?

4. Given $(X, \mathcal{M}, \mu), \mu(X) < \infty$, and let $f_n \to f$ pointwise on X, $f_n : X \to \mathbb{R}$ measurable. Assume that for each $\epsilon > 0$, there is a $\delta = \delta(\epsilon)$ such that $E \in \mathcal{M}$ and $\mu(E) \leq \delta$ implies that $\left|\int_E f_n d\mu\right| \leq \epsilon$. Show that $f_n \to f$ in L^1.

5. Let $(X, \mathcal{M}, \mu)$ be a measure space with $\mu(X) = 1$. Let $E_j \in \mathcal{M}, j = 1, \ldots, n$. If every $x \in X$ is in at least k of these sets, show that there exists $1 \leq j_0 \leq n$ such that $\mu(E_{j_0}) \geq \dfrac{k}{n}$.

6. Let $f \in L^1(I_0), f \geq 0$, and let for each positive integer n,
$$f_n(x) = \begin{cases} n, & f(x) \geq n \\ f(x), & f(x) < n \end{cases}.$$
Show that
$$\int_0^1 \log f_n \; dx \to \int_0^1 \log f \; dx.$$
Note that the integrals could be $-\infty$.

7. Let $A \subset \mathbb{R}$, A measurable, and show that $\forall r \in [0, |A|], \exists E \subset A$ with $|E| = r$. Can it be generalized to $\mathbb{R}^n$?

8. Evaluate the following limits and fully justify your answers:

   (a) $\lim_{n\to\infty} \int_0^n \left(1 - \dfrac{x}{n}\right)^n e^{x/2} dx$

   (b) $\lim_{n\to\infty} \int_0^\infty \dfrac{n}{e^x + n^2 x} dx$

9. Let $\{f_n\}$ be a sequence of nonnegative functions in $L^1([0,1])$ with the property that

$$\int_0^1 f_n(t)dt = 1 \text{ and } \int_{1/n}^1 f_n(t)dt \leq \frac{1}{n}$$

for all n. Define $h(x) = \sup_n f_n(x)$. Prove that $h \notin L^1([0,1])$.

10. Let $1 > \epsilon_j > 0, j = 1, 2, \ldots$. Show that $\sum \epsilon_j < \infty$ is necessary and sufficient so that $\sum \chi_{A_j}(x) < \infty$ a.e. whenever $\{A_j\}$ is a sequence of Borel sets in I_0 with $m(A_j) = \epsilon_j$.