(22 pts) **1.** Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

	Yes	No
If $y(t) = x(2t)$, is the system causal?	✓	causal depends on t
If $y(t) = (t+2)x(t)$, is the system causal?	✓	
If $y(t) = x(-t^2)$, is the system causal?	✓	
If $y(t) = x(t) + t - 1$, is the system memoryless?	✓	
If $y(t) = x(t^2)$, is the system memoryless?		✓
If $y(t) = x(t/3)$, is the system stable?	✓	
If $y(t) = tx(t/3)$, is the system stable?		✓
If $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, is the system stable?	✓	
If $y(t) = \sin(x(t))$, is the system time invariant?	✓	
If $y(t) = u(t) * x(t)$, is the system LTI?	✓	
If $y(t) = (tu(t)) * x(t)$, is the system linear?		✓

$x(t) \rightarrow \boxed{\quad}$

$y(t) = x(t_0 - t_0)$

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(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t+2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all $\sum$ signs disappear.)

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\, u(t-\tau+2)\, d\tau$$

$u(\tau) = 1 \quad \tau > 0$

$$= \int_{0}^{\infty} e^{-\tau} u(t-\tau+2)\, d\tau$$

Cute ✓

$u(t-\tau+2) = 1 \quad t-\tau+2 > 0$

$\tau < t+2$

$$= \int_{0}^{t+2} e^{-\tau}\, d\tau$$

$$= -e^{-\tau}\Big|_0^{t+2} = -e^{-(t+2)} - (-e^{0})$$

$$\boxed{y(t) = \left(-e^{-(t+2)} + 1\right) u(t+2)}$$

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(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} \left| \frac{3e^{jt}}{1+j} \right|^2 dt$$

$|e^{jt}| = 1$

$|3e^{jt}| = 3$

$|3e^{jt}|^2 = 3^2 = 9$

$$= \int_{-\infty}^{\infty} \frac{9}{(1+j)^2} \, dt$$

$(1+j)(1+j) = 1 + 2j - 1 = 2j$

$$= \int_{-\infty}^{\infty} \frac{9}{2j} \, dt = \frac{9}{2j} t \Big|_{-\infty}^{\infty} = \boxed{\infty}$$

$$P_\infty = \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \frac{1}{2T} \int_{-T}^{T} \left| \frac{3e^{jt}}{1+j} \right|^2 dt$$

$$= \frac{1}{2T} \int_{-T}^{T} \frac{9}{2j} \, dt = \frac{1}{2T} \left(\frac{9}{2j} t \Big|_{-T}^{T} \right) = \frac{1}{2T} \left(\frac{9T}{2j} - \frac{-9T}{2j} \right) = \frac{1}{2\cancel{T}} \frac{18\cancel{T}}{2j} = \frac{18}{4j}$$

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(15 pts) **4.** Compute the coefficients a_k of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}.$$

(Simplify your answer as much as possible.)

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\left(\frac{2\pi}{T}\right)t}\, dt \qquad T = 4 \qquad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a_k = \frac{1}{4}\left(\int_0^2 \sin(\pi t)\, e^{-jk\frac{\pi}{2}t}\, dt + \cancel{\int_2^4 0\, e^{-jk\frac{\pi}{2}t}\, dt}^{\,0} \right)$$

$a_0 =$ average of signal

average of $\sin(\pi t) = 0$

$\therefore \boxed{a_0 = 0}$

$$\sin(\pi t) = \frac{e^{j\pi t} - e^{-j\pi t}}{2j} = \frac{e^{j\pi t}}{2j} - \frac{e^{-j\pi t}}{2j}$$

$$\boxed{a_1 = \frac{1}{2j} \quad a_{-1} = -\frac{1}{2j}}$$

for general a_k

$$a_k = \frac{1}{8j}\int_0^2 e^{j\pi t} e^{-jk\frac{\pi}{2}t} - e^{-j\pi t} e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{8j}\int_0^2 e^{t(j\pi - jk\pi/2)} - e^{t(-j\pi - jk\pi/2)}\, dt$$

$$= \frac{1}{8j}\left(\frac{1}{j\pi - jk\frac{\pi}{2}} e^{t(j\pi - jk\frac{\pi}{2})} - \frac{1}{-j\pi - jk\frac{\pi}{2}} e^{t(-j\pi - jk\frac{\pi}{2})} \right)\Bigg|_0^2$$

5. A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

$$
\begin{array}{rcl}
\text{input} & & \text{output} \\
x_0[n] = \delta[n] & \rightarrow & y_0[n] = \delta[n-1], \\
x_1[n] = \delta[n-1] & \rightarrow & y_1[n] = 4\delta[n-2], \\
x_2[n] = \delta[n-2] & \rightarrow & y_2[n] = 9\delta[n-3], \\
x_3[n] = \delta[n-3] & \rightarrow & y_3[n] = 16\delta[n-4], \\
& \vdots & \\
x_k[n] = \delta[n-k] & \rightarrow & y_k[n] = (k+1)^2\delta[n-(k+1)] \text{ for any integer k.}
\end{array}
$$

(10 pts) **a)** Can this system be time-invariant? Explain.

5

No, a time-invariant system holds that if the input is shifted by t_0 then the resulting output is als shifted by t_0. This is not the case for this system.

be more precise.

(10 pts) **b)** Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n-1]$?

It can be shown that the step function $u[n]$ is a series of impulse functions:

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$$u[n] = \sum_{k=0}^{\infty} \delta(n-k)$$

and since if $x[n] = \delta[n]$ then $y[n] = \delta(n-$

$$x_1[n] = \sum_{k=0}^{\infty} \delta(n-k) \text{ will yield } \sum u[n-1]$$

$u[n-1]$