Fall 2007

(22 pts) **1.** Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

	Yes	No	
If $y(t) = x(2t)$, is the system causal?		X	$y(2) = x(4)$
If $y(t) = (t+2)x(t)$, is the system causal?	X		
If $y(t) = x(-t^2)$, is the system causal?		X	$y(-\frac{1}{2}) = x(\frac{-1}{4})$ depends on future
If $y(t) = x(t) + t - 1$, is the system memoryless?	X		
If $y(t) = x(t^2)$, is the system memoryless?		X	$y(2) = x(4)$
If $y(t) = x(t/3)$, is the system stable?	X		
If $y(t) = t x(t/3)$, is the system stable?		X	
If $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, is the system stable?		X	
If $y(t) = \sin(x(t))$, is the system time invariant?	X		
If $y(t) = u(t) * x(t)$, is the system LTI?	X		
If $y(t) = (tu(t)) * x(t)$, is the system linear?		X	

(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t+2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all $\sum$ signs disappear.)

$y(t) = x(t) * h(t)$

$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\, u(t+2-\tau)\, d\tau \qquad \tau > 0$

$= \int_{0}^{\infty} e^{-\tau} u(t+2-\tau)\, d\tau \qquad t+2-\tau > 0 \quad \tau < t+2$

$= \int_{0}^{t+2} e^{-\tau}\, d\tau$ if $\tau < t+2$, else 0

$= \left[-e^{-\tau}\right]\Big|_{0}^{t+2} u(t+2) = \left(-e^{-(t+2)} + 1\right) u(t+2)$

$= \left(1 - e^{-(t+2)}\right) u(t+2)$

(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E_\infty = \int_{-\infty}^{\infty} \left(\frac{3}{\sqrt{2}}\right)^2 = \frac{9}{2}t\Big|_{-\infty}^{\infty} = \boxed{\infty}$$

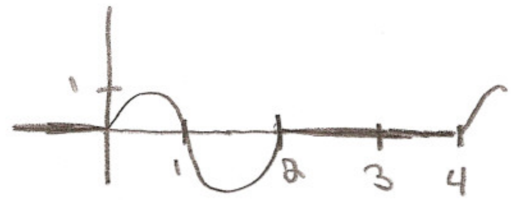
$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{9}{2}\,dt$$

$$P_\infty = \lim_{T\to\infty} \left(\frac{1}{2T}\right)\left(\frac{9}{2}t\Big|_{-T}^{T}\right)$$

$$P_\infty = \lim_{T\to\infty} \left(\frac{1}{2T}\right)\left(\frac{9}{2}\right)(2T) = \boxed{\frac{9}{2}}$$

(15 pts) **4.** Compute the coefficients a_k of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}.$$



(Simplify your answer as much as possible.)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(\frac{\pi}{2})t}$$

$$a_k = \frac{1}{T}\int_0^T x(t) e^{-jk(\frac{\pi}{2})t} dt$$

a_0 = average of signal $= \boxed{0}$

$$a_1 = \frac{1}{4}\int_0^4 x(t) e^{-j\frac{\pi}{2}t} dt = \frac{1}{4}\int_0^2 \sin(\pi t) e^{-j\frac{\pi}{2}t} dt$$

$$\sin(\pi t) = \frac{e^{j\pi t} - e^{-j\pi t}}{2j} \qquad a_1 = \frac{1}{8j}\int_0^2 e^{\frac{j\pi}{2}t} - e^{-\frac{3}{2}j\pi t} = \frac{1}{8j}\int_0^2 e^{\frac{j\pi}{2}t} - e^{-\frac{3}{2}j\pi t} e^{2\pi j t}$$

$$a_1 = \frac{1}{8j}\int_0^2 e^{\frac{j\pi}{2}t} - e^{\frac{j\pi}{2}t} dt = \frac{1}{8j}\int_0^2 0\,dt = \boxed{0}$$

$$a_2 = \frac{1}{4}\int_0^4 x(t) e^{-j\pi t} dt = \frac{1}{8j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right) e^{-j\pi t} dt = \frac{1}{8j}\int_0^2 -e^{-2j\pi t} dt = 1$$

$$a_2 = \frac{1}{8j}\int_0^2 -1\,dt = \frac{1}{8j}[-t]_0^2 = \boxed{\frac{-1}{4j}}$$

$$\to e^{-\frac{1}{2}j\pi t}$$

$$a_3 = \frac{1}{4}\int_0^4 x(t) e^{-j\frac{3\pi}{2}t} = \frac{1}{8j}\int_0^2 \sin(\pi t) e^{-j\frac{3\pi}{2}t} = \frac{1}{8j}\int_0^2 \left(e^{-j\frac{\pi}{2}t} - e^{-\frac{5}{2}j\pi t}\right) dt$$

$$a_3 = \frac{1}{4}\int_0^2 \left(e^{-j\frac{\pi}{2}t} - e^{-j\frac{\pi}{2}t}\right) dt = \boxed{0}$$

$$\boxed{\begin{aligned} a_0 &= 0 \\ a_1 &= 0 \\ a_2 &= \frac{-1}{4j} \\ a_3 &= 0 \end{aligned}}$$

5. A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

$$\begin{array}{rcl}
\text{input} & & \text{output} \\
x_0[n] = \delta[n] & \rightarrow & y_0[n] = \delta[n-1], \\
x_1[n] = \delta[n-1] & \rightarrow & y_1[n] = 4\delta[n-2], \\
x_2[n] = \delta[n-2] & \rightarrow & y_2[n] = 9\delta[n-3], \\
x_3[n] = \delta[n-3] & \rightarrow & y_3[n] = 16\delta[n-4], \\
& \vdots & \\
x_k[n] = \delta[n-k] & \rightarrow & y_k[n] = (k+1)^2\delta[n-(k+1)] \text{ for any integer k.}
\end{array}$$

(10 pts) **a)** Can this system be time-invariant? Explain.

$\boxed{To = 1}$

No

$\left.\begin{array}{l} \delta[n] \rightarrow \text{time shift} \rightarrow d[n-1] \rightarrow \text{system} \rightarrow 4\delta[n-2] \\ \delta[n] \rightarrow \text{system} \rightarrow d[n-1] \rightarrow \text{time shift } \delta[n-2] \end{array}\right\}$ Not Equal

(10 pts) **b)** Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n-1]$?

u[n]? not sure how to do this?