

The nice thing about complex sinewaves is that the phase factors out as part of a complex amplitude

$$x(t) = A e^{-j(\omega t + \theta)} = A \underline{e^{-j\theta}} \cdot \underline{e^{j\omega t}}$$

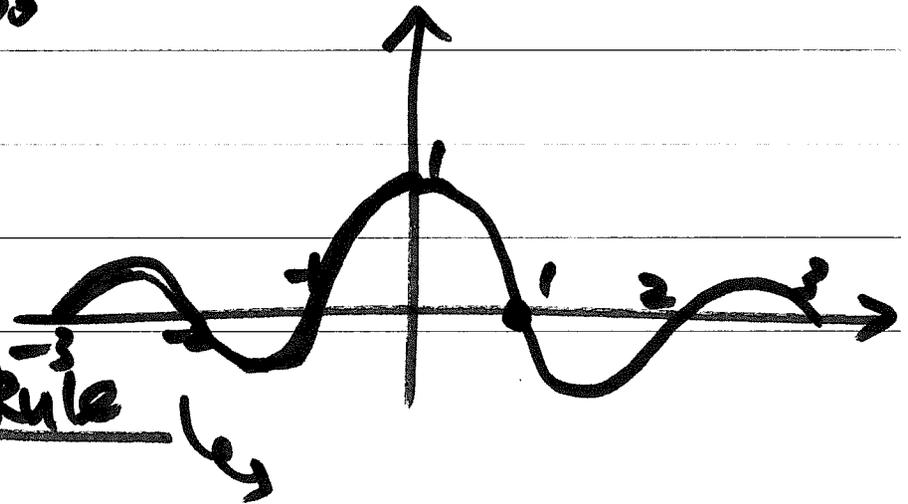
This is not true for real-valued sinewaves.

6) Sinc func. (Important in many engr / physics fields)

$$\text{sinc}(t) = \frac{\sin(\pi t) \rightarrow 0}{\pi t \rightarrow 0}$$

eg. $\text{sinc}(1) = \frac{\sin(\pi)}{\pi} = 0$

$\text{sinc}(0) \Rightarrow$ L'Hospital's Rule



$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

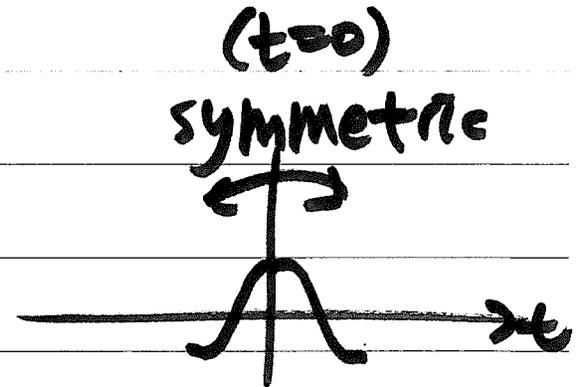
"L'Hospital's"

$$\lim_{t \rightarrow 0} \frac{\sin(\pi t)}{\pi t} = \frac{\pi \cdot \cos(\pi t)}{\pi} \Big|_{t=0} = \frac{\pi}{\pi} = 1$$

1) Even and Odd signals

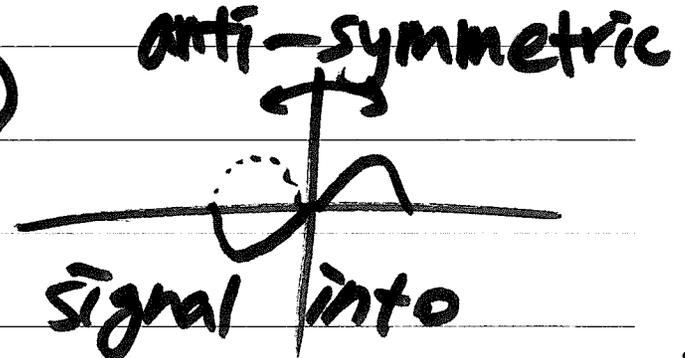
- Even : $x(-t) = x(t)$

e.g. $\cos(\omega \cdot t)$



- Odd : $x(-t) = -x(t)$

e.g. $\sin(\omega \cdot t)$



→ can decompose arbitrary signal into

even and odd parts : $x(t) = x_e(t) + x_o(t)$

$$x_e(t) = \text{Even} \{x(t)\} = \frac{1}{2} \{x(t) + x(-t)\}$$

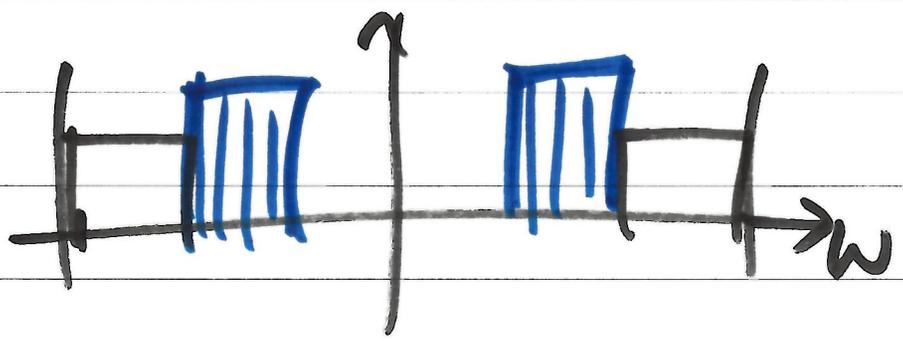
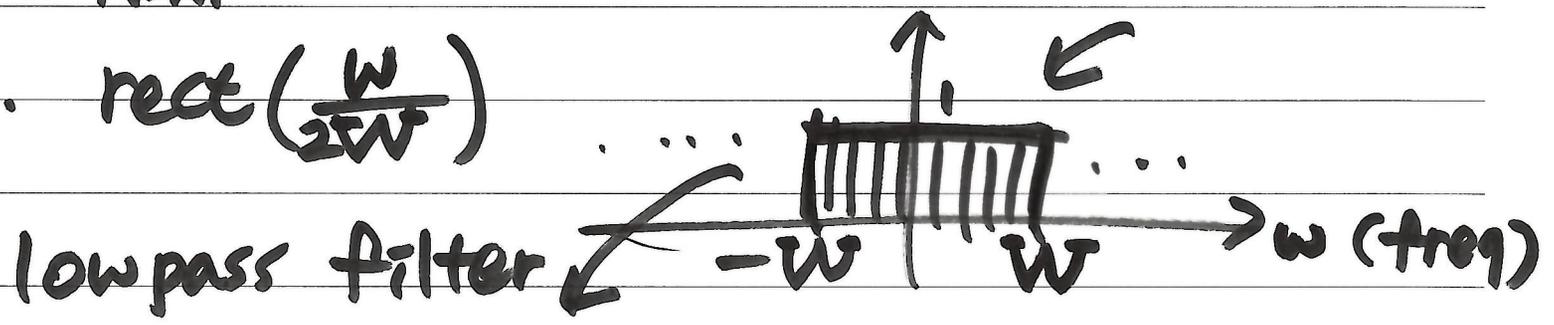
$$x_o(t) = \text{Odd} \{x(t)\} = \frac{1}{2} \{x(t) - x(-t)\}$$

~~$x(t)$~~ ~~x~~
 $f(t) \leftrightarrow f(x)$

e.g. could be position $x \rightarrow \text{sinc}(x)$

later, they will ^{also} be used as function of
 freq. ω . \rightarrow gives very different inter-
 pretation.

e.g. $\text{rect}(\frac{\omega}{2W})$



1-2. Discrete-Time Signals (DT signals) (4)

1) Typically obtained by sampling an analog signal at equi-spaced instants in time.

$$\boxed{\begin{array}{l} \swarrow \text{DT signal} \quad \swarrow \text{CT signal} \\ \underline{x[n]} = \underline{x(t)} \Big|_{t=nT_s} = x(nT_s) \end{array}}, \quad -\infty < n < \infty \\ n: \text{integer}$$

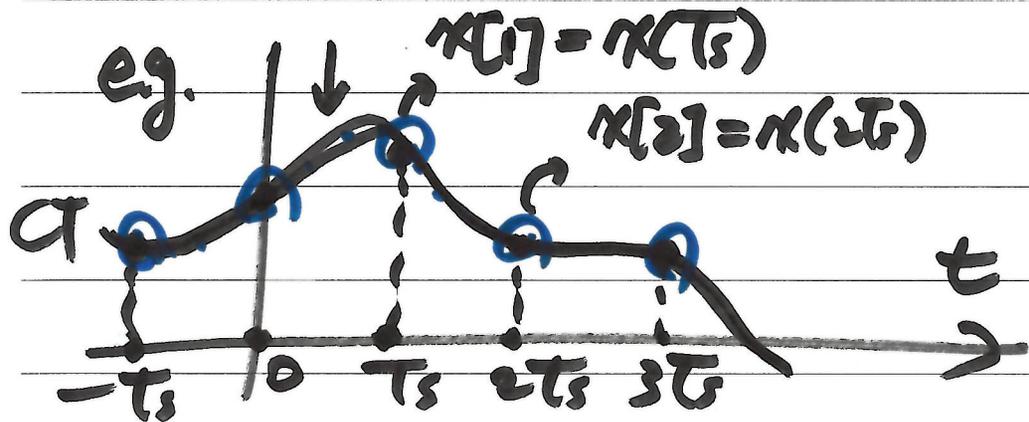
$$\begin{array}{l} \langle \text{DT} \rangle \quad \downarrow \quad \langle \text{CT} \rangle \\ x[-1] = x(-T_s) \\ x[0] = x(0) \\ x[1] = x(T_s) \\ x[2] = x(2T_s) \\ \vdots \end{array} \left. \vphantom{\begin{array}{l} \langle \text{DT} \rangle \\ \langle \text{CT} \rangle \end{array}} \right\} \begin{array}{l} \text{the samples of a CT signal} \\ \text{are stored in an array} \\ \rightarrow x[n] \\ \text{(n is like an index location)} \end{array}$$

* DT : discrete-time
CT : continuous-time

• $x[n + \frac{1}{2}]$ is meaningless \rightarrow doesn't make sense

$$= x\left(\left(n + \frac{1}{2}\right)T_s\right) = x\left(nT_s + \frac{T_s}{2}\right)$$

We have a sample at nT_s and $(n+1)T_s$ but not at halfway in between.



$x\left[1 + \frac{1}{2}\right] = x\left[\frac{3}{2}\right]$
doesn't make sense.

* sampling rate $\uparrow = \frac{1}{T_s} \downarrow$ (num. of samples per sec)

2) DT Sine waves " \forall : for all "

$$x[n] = e^{j\omega_0 n}, \quad n : \text{integer}$$

• suppose DT sine waves was obtained by sampling a CT sine waves.

$$\begin{aligned} x[n] &= \underline{e^{j\omega_0 t}} \Big|_{t=nT_s} = e^{j2\pi f_0 t} \Big|_{t=nT_s} \\ &= e^{j(2\pi f_0 T_s) n} \\ &= \underline{e^{j\omega_0 n}}, \quad \omega_0 = 2\pi f_0 T_s \end{aligned}$$

• because n is an integer, DT sine waves are very diff. from CT sine waves.

* diff. : different

(6)

Two major differences (CT vs DT sinewaves) ①

(1) CT sinewaves are unique as long as freq. are diff.

→ but it's not true for DT sinewaves

↳ consider $e^{j(\omega + l \cdot 2\pi)n}$, l, n : integers

(recall $e^{j\theta} = \cos\theta + j\sin\theta$
● $e^{j2\pi} = \underbrace{\cos(2\pi)}_{=1} + j\underbrace{\sin(2\pi)}_{=0} = 1$)

$$\begin{aligned} \text{Thus, } e^{j(\omega + l \cdot 2\pi)n} &= e^{j\omega n} \cdot e^{j2\pi ln} \\ &= e^{j\omega n} \underbrace{\left(\frac{e^{j2\pi}}{=1} \right)^{ln}}_{=1} = \underline{e^{j\omega n}} \end{aligned}$$

→ Any two DT sinewaves whose freq. are separated by an integer multiple of 2π are the same.

(2) CT sinewaves are always periodic
→ but it's not always true for DT sinewaves.

↳ the period for a DT sinewave has to be an integer $N \Rightarrow \underline{x[n] = x[n+N]}$, $\forall n$

$$e^{j\omega_0(n+N)} = e^{j\omega_0 n} \cdot e^{j\omega_0 N}$$

In order to be periodic

$$= e^{j\omega_0 n} \cdot e^{j m \cdot 2\pi} \rightarrow \underline{\omega_0 N = m \cdot 2\pi}$$
, m : Integer

$$\rightarrow \left(\frac{\omega_0}{2\pi}\right) = \frac{m}{N}$$

Integers

Must be rational for DT sinewave to be periodic

IF this condition holds:

$$e^{j\omega_0 n} = e^{j 2\pi \frac{m}{N} (n+N)} = e^{j 2\pi \frac{m}{N} n}$$

