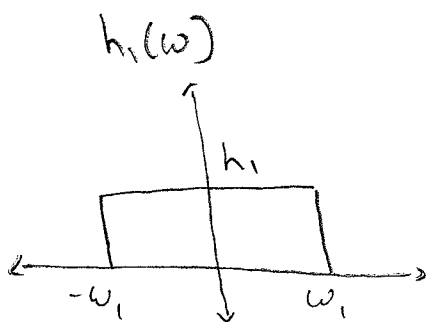
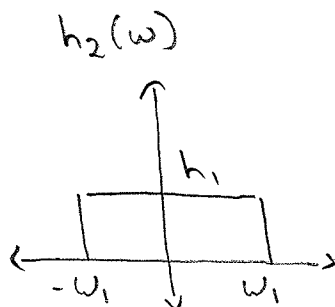


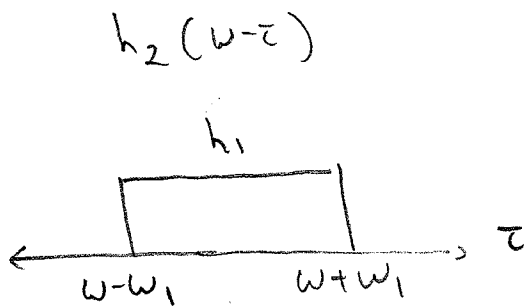
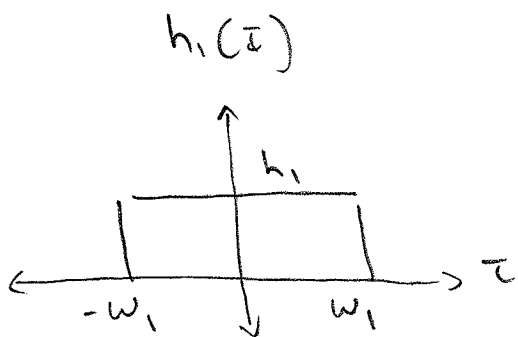
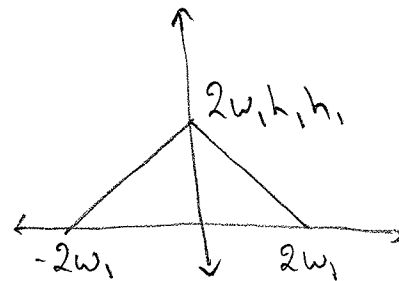
Bonus 6



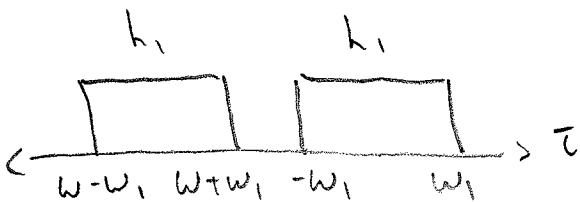
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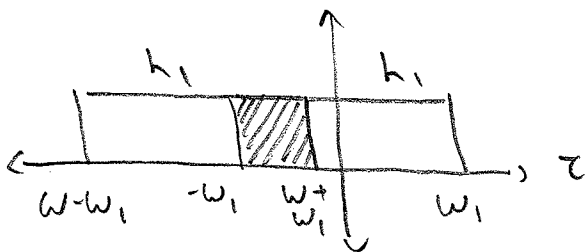
Interval I -



$$\begin{aligned} \omega + \omega_1 &< -\omega_1 \\ \omega &< -\omega_1 - \omega_1 \\ \omega &< -2\omega_1 \end{aligned}$$

$$y(\omega) = 0$$

Interval II -

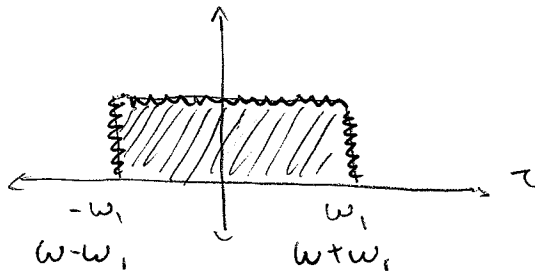


$$\begin{aligned} \omega - \omega_1 &< -\omega_1 & \omega + \omega_1 &> -\omega_1 \\ \omega &< -\omega_1 + \omega_1 & \omega &> -\omega_1 - \omega_1 \\ \omega &< 0 & \omega &> -2\omega_1 \end{aligned}$$

$$\text{i.e. } -2\omega_1 < \omega < 0$$

$$\begin{aligned} y(\omega) &= \int_{-\omega_1}^{\omega + \omega_1} (h_1)^2 d\tau = h_1 h_1 \int_{-\omega_1}^{\omega + \omega_1} d\tau \\ &= h_1 h_1 \left[\tau \Big|_{-\omega_1}^{\omega + \omega_1} \right] = h_1 h_1 \left[\omega + 2\omega_1 \right] \end{aligned}$$

Interval III



$$\omega - \omega_1 = -\omega_1$$

$$\omega = 0$$

$$\omega_1 = \omega + \omega_1$$

$$\omega = 0$$

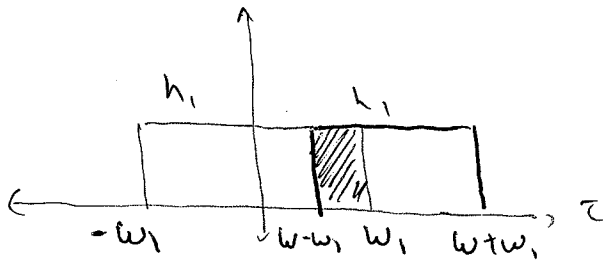
i.e. At $\omega = 0$

$$y(\omega) = h_1 h_1 \int_{\omega - \omega_1}^{\omega + \omega_1} 1 d\tau$$

$$= h_1 h_1 \left[\tau \Big|_{\omega - \omega_1}^{\omega + \omega_1} \right]$$

$$= 2\omega_1 h_1 h_1 = 2\omega_1 (h_1^2)$$

Interval IV



$$\omega - \omega_1 > -\omega_1$$

$$\omega > -\omega_1 + \omega_1$$

$$\omega > 0$$

$$\omega - \omega_1 < \omega_1$$

$$\omega < \omega_1 + \omega_1$$

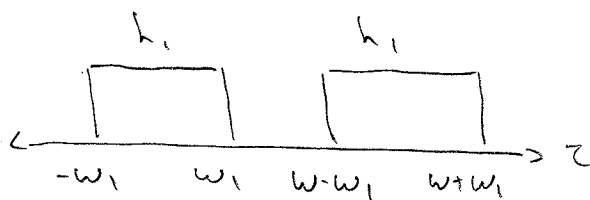
$$\omega < 2\omega_1$$

i.e. $0 < \omega < 2\omega_1$

$$y(\omega) = \int_{\omega - \omega_1}^{\omega_1} h_1 h_1 d\tau = (h_1 h_1) \left[\tau \Big|_{\omega - \omega_1}^{\omega_1} \right]$$

$$= h_1^2 [2\omega_1 - \omega]$$

Interval V



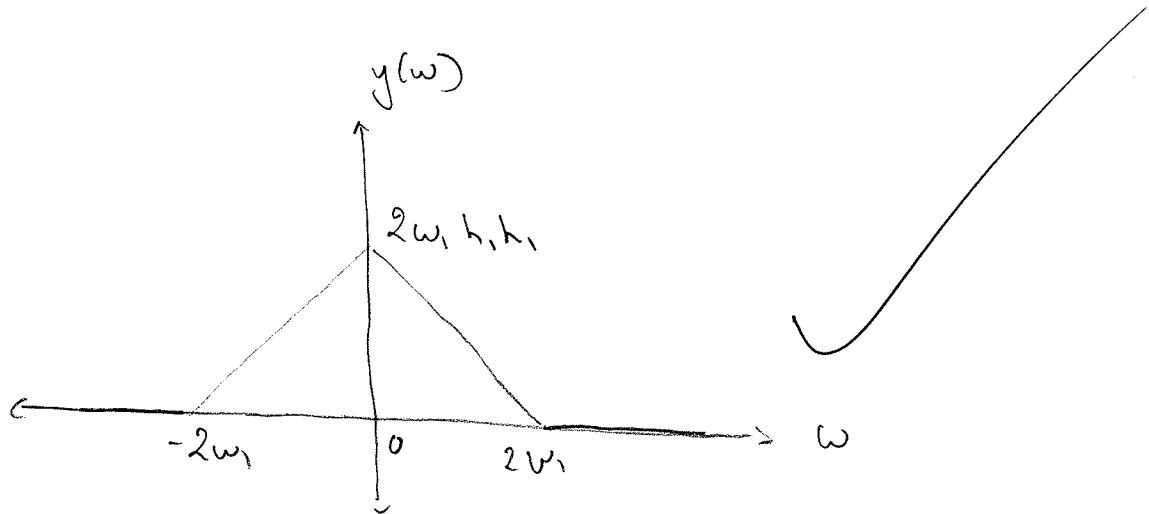
$$\omega_1 < \omega - \omega_1$$

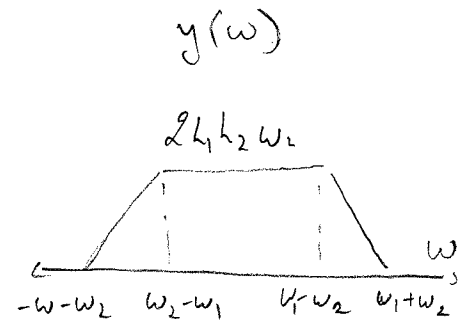
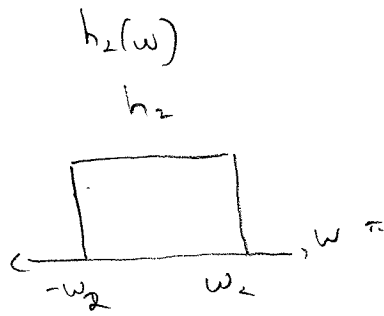
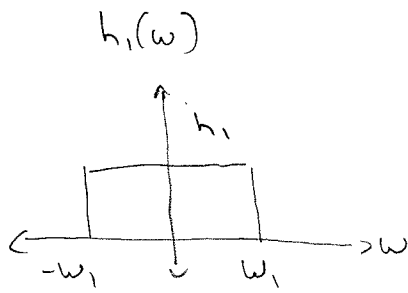
$$2\omega_1 < \omega$$

$$\omega > 2\omega_1$$

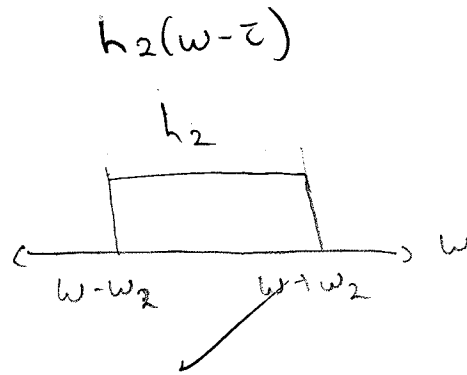
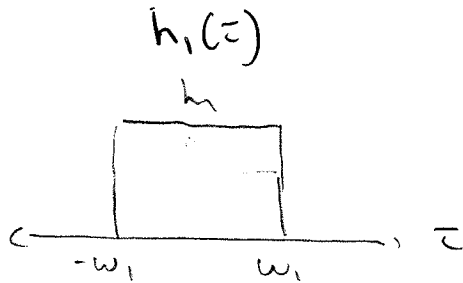
$$y(\omega) = 0$$

$$\therefore y(\omega) = \begin{cases} 0 & \omega < -2\omega_1 \\ h_1^2 [\omega + 2\omega_1] & -2\omega_1 < \omega < 0 \\ 2\omega_1 h_1^2 & \omega = 0 \\ h_1^2 [2\omega_1 - \omega] & 0 < \omega < 2\omega_1 \\ 0 & \omega > 2\omega_1 \end{cases}$$

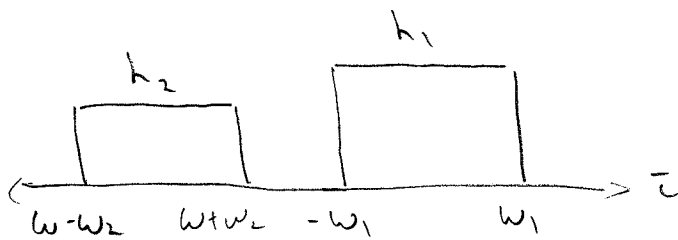




$w_1 > w_2$



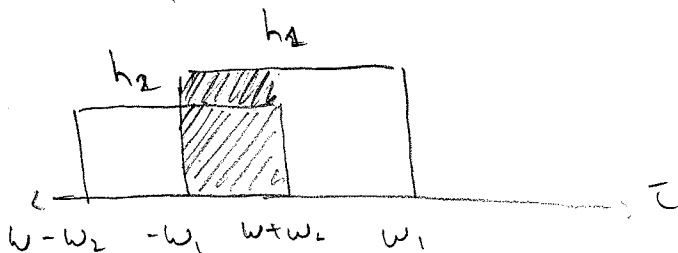
Interval I -



$w+w_2 < -w_1$
 $w < -w_1 - w_2$

$y(w) = 0$

Interval II -



$w-w_2 < -w_1$
 $w < -w_1 + w_2$

$w+w_2 > -w_1$
 $w > -w_1 - w_2$
 $w > -(w_1 + w_2)$

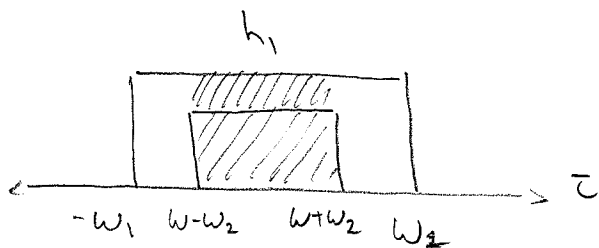
i.e. $-(w_1 + w_2) < w < -w_1 + w_2$

$$y(w) = \int_{-w_1}^{w+w_2} h_1 h_2 d\tau$$

$$= (h_1 h_2) \left[\tau \Big|_{-w_1}^{w+w_2} \right]$$

$$= h_1 h_2 (w + w_2 + w_1)$$

Interval III -

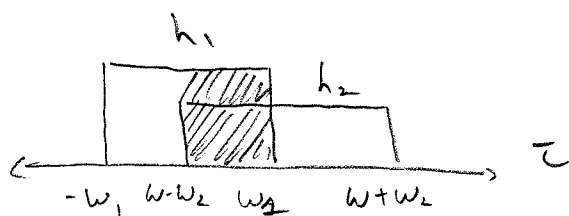


$$y(\omega) = \int_{\omega-w_2}^{\omega+w_2} h_1 h_2 \mathbb{1} d\tau = h_1 h_2 (2w_2)$$

$$\begin{aligned} \omega - w_2 > -w_1 & & \omega + w_2 \leq w_1 \\ \omega > -w_1 + w_2 & & \omega \leq w_1 - w_2 \end{aligned}$$

i.e. $-w_1 + w_2 < \omega < w_1 - w_2$

Interval IV -

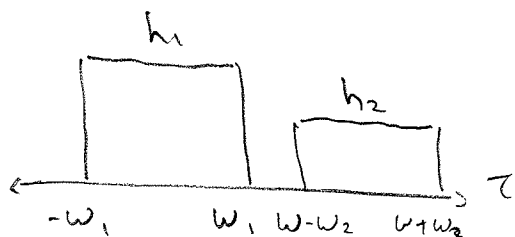


$$y(\omega) = \int_{\omega-w_2}^{\omega_1} h_1 h_2 \mathbb{1} d\tau = h_1 h_2 (\omega_1 - \omega + w_2)$$

$$\begin{aligned} \omega - w_2 < \omega_1 & & \omega + w_2 > \omega_1 \\ \omega < \omega_1 + w_2 & & \omega > \omega_1 - w_2 \end{aligned}$$

$\omega - w_2 < \omega < \omega_1 + w_2$

Interval V -



$$\begin{aligned} \omega - w_2 > \omega_1 \\ \omega > \omega_1 + w_2 \end{aligned}$$

$$y(\omega) = 0$$

$$y(\omega) = \begin{cases} 0 & \omega < -w_1 - w_2 \quad \checkmark \\ h_1 h_2 (\omega + w_2 + w_1) & -(w_1 + w_2) < \omega < -w_1 + w_2 \\ h_1 h_2 (2w_2) \quad \checkmark & -w_1 + w_2 < \omega < w_1 - w_2 \\ h_1 h_2 (\omega_1 - \omega + w_2) & \omega - w_2 < \omega < \omega_1 + w_2 \\ 0 & \omega > \omega_1 + w_2 \quad \checkmark \end{cases}$$

$y(\omega)$

