1. Let $f : X \to Y$ a continuous bijection with X, Y both metric spaces. If X is compact, show f^{-1} is also continuous. Does the result hold without compactness?

2. Let $f : \mathbb{R} \to \mathbb{R}$ such that $\sum a_n = a \implies \sum f(a_n) = f(a)$. Show $f(x) = cx$ for a fixed constant c.

3. Does there exist a non-continuous function $f : \mathbb{Z} \to \mathbb{R}$?

4. Let X and Y be metric spaces and $f : X \to Y$. Show f is continuous if and only if for every sequence $\{x_n\} \subset X$ with $x_n \to x \in X$, $f(x_n) \to f(x)$.

5. Let $f : X \to Y$ a continuous function with X, Y both metric spaces. True or false (with proof/ counter-example):

   (a) $\forall G$ open in $Y, f^{-1}(G)$ is open in X

   (b) $\forall G$ open in $X, f(G)$ is open in Y

   (c) $\forall F$ closed in $Y, f^{-1}(F)$ is closed in X

   (d) $\forall F$ closed in $X, f(F)$ is closed in Y

   (e) $\forall K$ compact in $Y, f^{-1}(K)$ is compact in$X$

   (f) $\forall K$ compact in $X, f^(K)$ is compact in Y