10/16/09

ECE 495N, Fall'09 GRIS 280, MWF 1130A – 1220P

HW#5: Due Friday Oct.23 in class.

Useful relation: $[h(\vec{k})] = \sum_{m} [H_{nm}] \exp(i\vec{k}.(\vec{d}_m - \vec{d}_n))$ *Bandstructure*

Problem 1: Consider an infinitely long linear 1-D lattice (lattice constant: a) with one s-orbital per atom (assumed orthogonal) and having a site energy of E_0, so that the Hamiltonian looks like

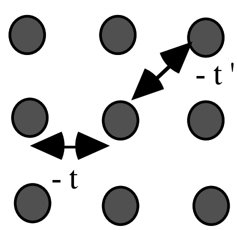
$$H = \begin{bmatrix} \varepsilon & te^{i\varphi} & 0 & 0 & \cdots \\ te^{-i\varphi} & \varepsilon & te^{i\varphi} & 0 & \cdots \\ 0 & te^{-i\varphi} & \varepsilon & te^{i\varphi} & \cdots \\ \cdots & \cdots & \cdots & & \end{bmatrix}$$

Find the dispersion relation E(k).

Problem 2: The $E(k_X,k_Y)$ relation for some solids is written in the form

$$E = E_0 - 2V(\cos k_X a + \cos k_y a + 2\alpha \cos k_X a \cos k_y a)$$

where α is a dimensionless number. How would you choose the nearest neighbor and next nearest neighbor overlap matrix elements in a square lattice of side 'a' so as to correspond to this dispersion relation ?



Nearest neighbor overlap : - t

Next nearest neighbor overlap : - t'

Problem 3: Suppose electrons are confined to a two-dimensional layer having an $\varepsilon(\vec{k})$ relation of the form

$$\varepsilon(\vec{k}) = \hbar^2(k_x{}^2 + k_y{}^2)/2m$$

How would you write the energies of the subbands if the layer is in the form of a small cylinder with a circumference of W in the y-direction?