(continue)

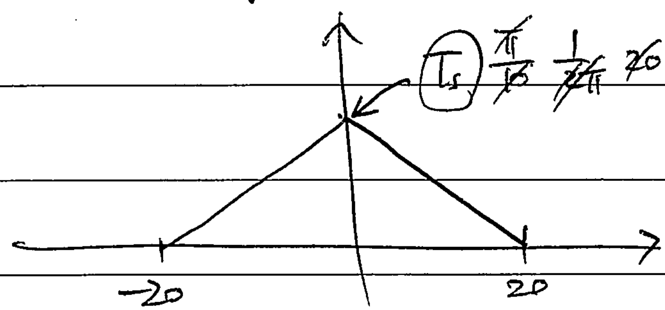
$$x[n] = x_a(nT_s) \quad \text{where} \quad x_a(t) = T_s \frac{\pi}{10}\left(\frac{\sin 10t}{\pi t}\right)^2$$

$$\text{and} \quad T_s = \frac{2\pi}{30} \qquad (\omega_s = 30)$$

$$h[n] = \frac{2\sin\left(\frac{\pi}{6}n\right)}{\pi n}\cos\left(\frac{5\pi}{6}n\right)$$

a) Any aliasing ?

First, plot $|X_a(\omega)|$



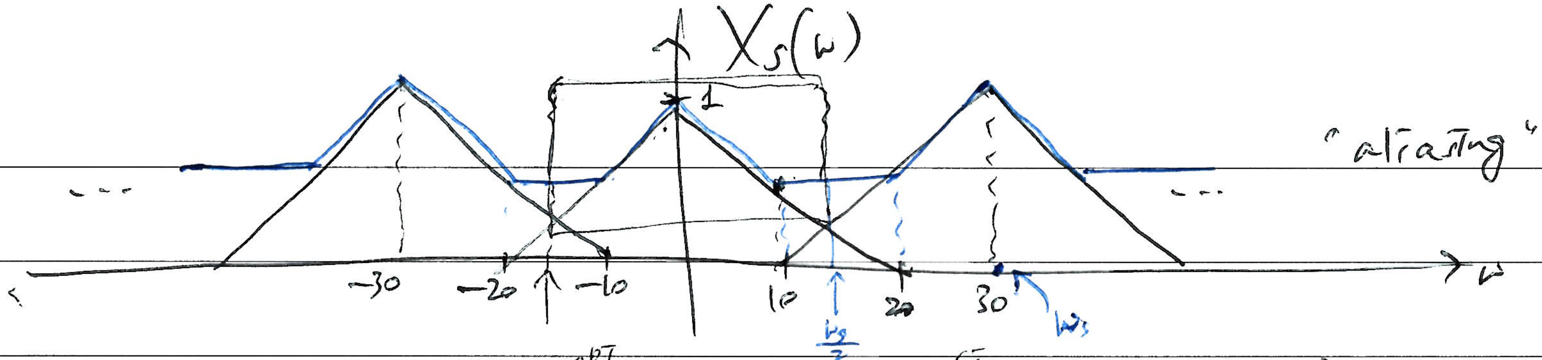
max freq $(\omega_M) = 20$

$\omega_s < 2\omega_M = 2 \cdot 20$
$(=30)$

$\Rightarrow$ there is an aliasing.
(not a perfect reconstruction)

b) plot $|X(\omega)|$: DTFT of $x[n]$

$$X_s(\omega) = \mathcal{F}\left\{\sum_n x_a(nT_s)\,\delta(t - nT_s)\right\}$$



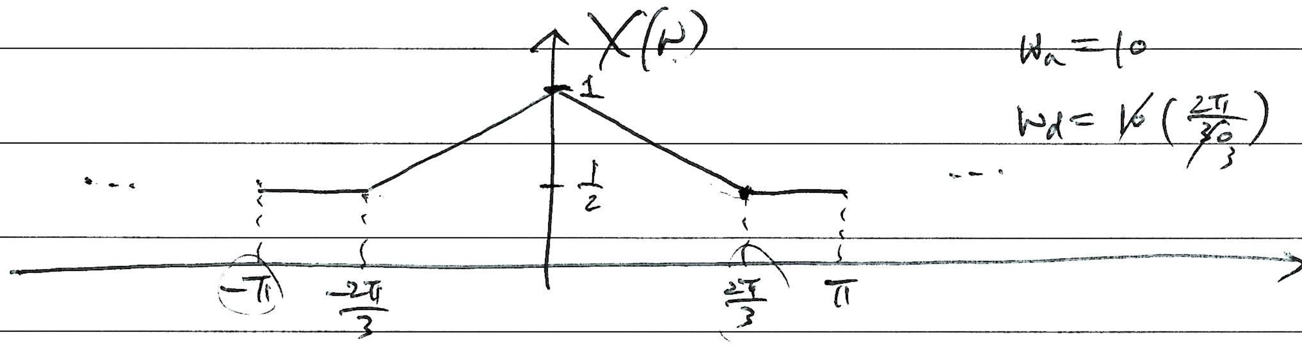
"aliasing"

Finally DTFT $X(\omega) = X_s(F_s \omega) = X_s\left(\frac{\omega}{T_s}\right) \to \frac{2\pi}{30}$

⇒ map to digital freq via "$\omega_d = \omega_a T_s$"

⇒ compress by sampling rate so that

$-\frac{\omega_s}{2} < \omega_a < \frac{\omega_s}{2}$ mapped to $-\pi < \omega < \pi$

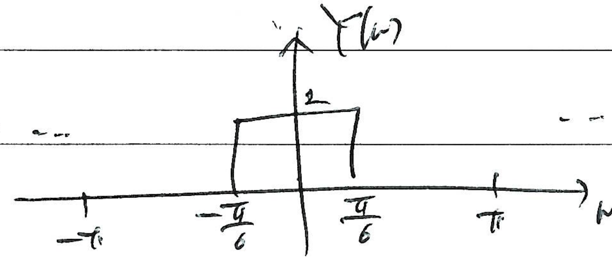


$\omega_a = 10$

$\omega_d = 10\left(\frac{2\pi}{30}\right) = \frac{2\pi}{3}$

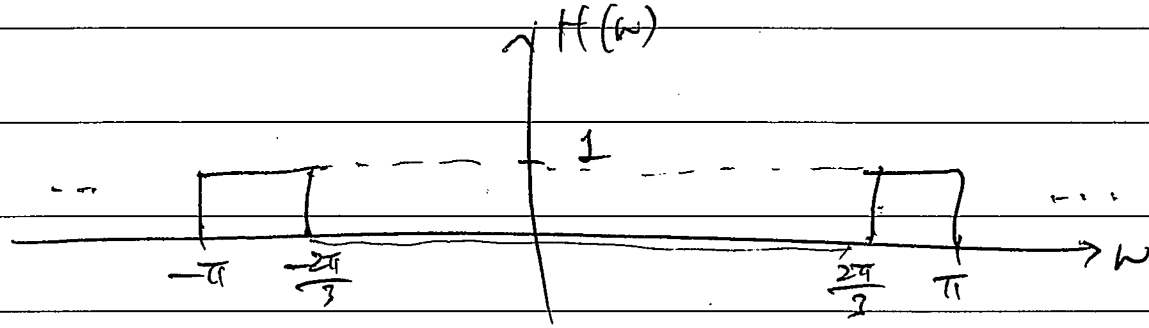
(c) Plot $|Y(\omega)|$

$\frac{\sin\left(\frac{\pi}{6}n\right)}{\pi n} \xleftrightarrow{\text{DTFT}}$



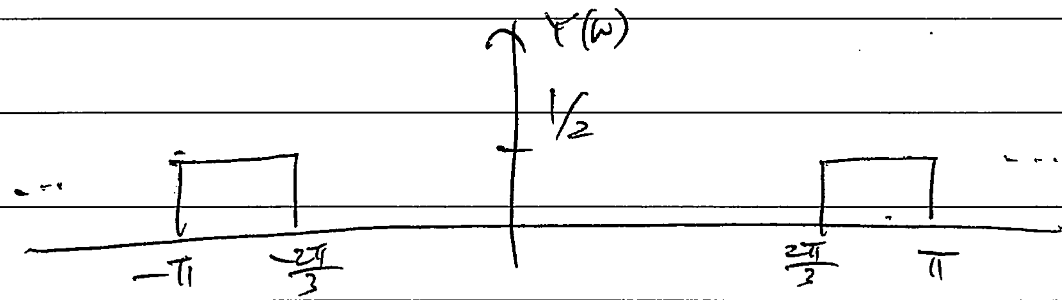
since $\cos\left(\frac{5\pi}{6}n\right) = \frac{1}{2}e^{j\frac{5\pi}{6}n} + \frac{1}{2}e^{-j\frac{5\pi}{6}n}$

$$H(\omega) = \text{DTFT}\left\{ \frac{2\sin\left(\frac{\pi}{6}n\right)}{\pi n} \cos\left(\frac{5\pi}{6}\right)\right\}$$



(period $= 2\pi$)

since $Y(\omega) = H(\omega)\,X(\omega)$



(d) $\sum_n y^2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y(\omega)|^2\, d\omega = \frac{1}{\cancel{2}\pi}\,\cancel{2}\,\underbrace{\left(\frac{1}{2}\right)^2}_{\text{height}}\,\underbrace{\frac{\cancel{\pi}}{3}}_{\text{width}} = \frac{1}{12}$

Chap 10. Z-Transform

ZT : $X(z) = \sum_{n=-\infty}^{\infty} x[n] \, \underline{z^{-n}}$

DTFT : $X(w) = \sum_{n=-\infty}^{\infty} x[n] \, \underline{e^{-jwn}}$

"$z = e^{jw}$"

$$X(z) = \mathcal{Z}\{x[n]\}$$

$$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$$

$$x[n] = \mathcal{Z}^{-1}\{X(z)\}$$

- Linearity : $\mathcal{Z}\{a_1 x_1[n] + a_2 x_2[n]\} = a_1 X_1(z) + a_2 X_2(z)$
- Time-shift : $x[n-n_0] \xleftrightarrow{\mathcal{Z}} z^{-n_0} X(z)$
- Convolution : $y[n] = \underline{x[n] * h[n]} \xleftrightarrow{\mathcal{Z}} Y(z) = X(z) H(z)$

$$H(z) = \frac{Y(z)}{X(z)}$$ given LTI system

Region of Convergence

- ZT pair : $a^n u[n] \overset{Z}{\longleftrightarrow} \dfrac{z}{z-a}$, $|z| > |a|$

- Time-shift property dictates.

$a^{n-1} u[n-1] \overset{Z}{\longleftrightarrow} z^{-1} \dfrac{z}{z-a} = \dfrac{1}{z-a}$. ROC is same as above.

Comment: Thus, one could use same rules/ properties for partial fraction expansion that you learned for LT.

$e^{at} u(t) \overset{\mathcal{L}}{\longleftrightarrow} \dfrac{1}{s-a}$ vs $a^{n-1} u[n-1] \overset{Z}{\longleftrightarrow} \dfrac{1}{z-a}$

<CT signal> <DT signal>