1. $x, y \in \mathbb{R}^k$

$$|x| = |x-y+y| \le |x-y| + |y| \qquad \Delta\text{-ineq}$$

so $|x| - |y| \le |x-y|$

By symmetry $||x| - |y|| \le |x-y|$. ▨

2. Let $\{F_\alpha\}_{\alpha \in A}$ be a family of compact sets.

$$\left(\bigcap_{\alpha\in A} F_\alpha\right)^c = \bigcup_{\alpha \in A} F_\alpha^c \qquad \text{(De Morgan)}$$

Which is open as $F_\alpha^c$ is open $\forall \alpha$.

$\Rightarrow \bigcap_\alpha F_\alpha$ is closed. ▨

3. $x, y \in \mathbb{R}^k \qquad x = (x_1, \ldots, x_k) \qquad y = (y_1, \ldots, y_k)$

(a) $d(x,y) = |x - 2y|$ not a metric since $d(1,1) = -1 \ne 0$.

(b) $d(x,y) = \max_i |x_i - y_i|$ is a metric.

$d(x,y) = 0 \iff x_i = y_i \ \forall i \iff x = y$ ✓

$d(x,y) \ge 0$ ✓

Need to show that $d(x,y) \le d(x,z) + d(z,y) \ \forall x, y, z \in \mathbb{R}^k$.

$d(x,y) = \max_i |x_i - y_i|$, so $\exists$ some $s \in \{1, \ldots, k\}$

$\ni d(x,y) = |x_s - y_s|$. Now we have

$|x_s - y_s| \le |x_s - z_s| + |z_s - y_s| \le \max_i |x_i - z_i| + \max_j |z_j - y_j| = d(x,z) + d(z,y)$

▨