Quiz 10
ECE302, Division II
Spring 2013

Slh

Student Name:_____

Grader Name:_____

Instructions: This is a traditional, non-collaborative quiz. No cheating, please!

Question: A random variables X has the following probability density function:

$$f_X(x) = Ce^{-\frac{1}{2}\frac{(x+3)^2}{9}}$$

where C is a constant. What is the expected value of X? (Justify your answer mathematically.)

HINT: Guess the answer m, and then compute E(X-m).

Answer:

$$E(X+3) = \int_{-\infty}^{\infty} (x+3) f_X(x)\,dx$$

$$= \int_{-\infty}^{\infty} (x+3)\, C e^{-\frac{1}{2}\frac{(x+3)^2}{9}}\,dx, \quad \text{let } u = x+3,\ du = dx$$

$$= C\int_{-\infty}^{\infty} u\, e^{-\frac{1}{2}\frac{u^2}{9}}\,du$$

$$= C\left[\int_{-\infty}^{0} u e^{-\frac{1}{2}\frac{u^2}{9}}\,du + \int_{0}^{\infty} u\, e^{-\frac{1}{2}\frac{u^2}{9}}\,du\right]$$

$v = -u$ in 1st int.

$$= C\left[-\int_{0}^{\infty} v e^{-\frac{1}{2}\frac{v^2}{9}}\,dv + \int_{0}^{\infty} u e^{-\frac{1}{2}\frac{u^2}{9}}\,du\right] = 0$$

$$\Rightarrow \quad E(X) + 3 = 0$$

$$\Rightarrow \quad E(X) = -3$$

Score: / 15