Midterm #3
Th. Nov 29
7-8pm

Midterm Examination 2
ECE 301
Division 3, Fall 2007
Instructor: Mimi Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. You have 50 minutes to complete the 5 questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
3. This booklet contains 10 pages. The last four pages contain a table of formulas and properties. You may tear out these pages **once the exam begins**. TABLE USE RULES: You may use any fact contained in the table without justification. Simply write the number of the corresponding table item to indicate which fact you are using from the table. If you use a non-trivial fact that is *not* contained in the table, you must justify (i.e., prove) it in order to get full credit.
4. This is a closed book exam. Calculators, cell phones, and i-pods are strictly forbidden.

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Itemized Scores	
Problem 1:	15
Problem 2:	9
Problem 3:	3
Problem 4:	10
Problem 5:	3
Total:	40

The mean is failing.

15

D.T.

(15 pts) **1.** Using the definition of the Fourier transform (*not* the table of Fourier transform pairs), compute the Fourier transform of the DT signal:

$$x[n] = \left(\frac{1}{2j}\right)^{|n|}.$$

$$\mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad \text{by (25)}$$

$$\mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2j}\right)^{|n|} e^{-j\omega n}$$

$$\mathcal{X}(\omega) = \sum_{n=1}^{\infty} \left(\frac{1}{2j}\right)^{n} e^{-j\omega n} + \underset{n=0}{1} + \sum_{n=-\infty}^{-1} \left(\frac{1}{2j}\right)^{|n|} e^{-j\omega(-n)}$$

Let $k=-n$ & flip bounds

$$\mathcal{X}(\omega) = \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{2j} e^{-j\omega}\right)^{n}} + 1 + \sum_{k=1}^{\infty} \left(\frac{1}{2j} e^{j\omega}\right)^{k}$$

by (user Proof #1)

$$\boxed{\mathcal{X}(\omega) = \frac{\frac{1}{2j} e^{-j\omega}}{1-\frac{1}{2j} e^{-j\omega}} + 1 + \frac{\frac{1}{2j} e^{j\omega}}{1-\frac{1}{2j} e^{j\omega}}}$$

(user Proof #1)

$$\sum_{n=1}^{\infty} \alpha^n \xrightarrow[n=\Psi+1]{\text{let } \Psi=n-1} \sum_{\Psi=0}^{\infty} \alpha^{\Psi+1} = \alpha \sum_{\Psi=0}^{\infty} \alpha^{\Psi}$$

$\not 9$ 9

C.T.

(20 pts) **2.** The Frequency response of a continuous-time LTI system is

$$H(j\omega) = \mathcal{H}(\omega) = \frac{1}{j\omega + 2}.$$

Use the convolution property of the Fourier transform to determine the response $y(t)$ when the input is $x(t) = e^{-|t|}$.

$x(t) = e^{-|t|}$

$\mathcal{X}(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ by (2)

$\mathcal{X}(\omega) = 2\int_0^{\infty} e^{-t} e^{-j\omega t} dt$

$\mathcal{X}(\omega) = 2\int_0^{\infty} e^{-t-j\omega t} dt = 2\int_0^{\infty} e^{-t(1+j\omega)} dt$

new approach

$x(t) = e^{-|t|}$

by (7)

$\mathcal{X}(\omega) = \cancel{\frac{1}{1+j\omega}}$ -2

-2

-2

$Y(\omega) = H(\omega) \cdot X(\omega)$

$Y(\omega) = \frac{1}{j\omega+2} \cdot \frac{1}{1+j\omega} = \frac{A}{2+j\omega} + \frac{B}{1+j\omega}$ -3

by (7) $\boxed{y(t) = Ae^{-2t} + Be^{-t}}$

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(15 pts) **3.** True/False? The Fourier transform of a DT signal $x[n]$ is a periodic function, no matter what $x[n]$ is. (Justify your answer.)

$\boxed{\text{True}}$

$$\mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad \text{by (25)}$$

$x[n]$ ~~terms~~ will allways have a $e^{-j\omega n}$ co-efficent, which by Eulers Identities is periodic (sinusodial)

w/ period 2π for any

So $\mathcal{X}(\omega)$ will allways be periodic

Look AT I.D.T.F.T.

$$x[n] = \frac{1}{2\pi}\int_{2\pi} \mathcal{X}(\omega) e^{j\omega n} d\omega \quad \text{by (26)}$$

The Signal is reconstructed using only one 2π period.

10

C.T.

(10 pts) **4.** A continuous-time LTI system has frequency response

$$H(j\omega) = \frac{j\omega + 4}{(j\omega+2)(j\omega+3)}.$$

Derive a differential equation representing this system. (Use the properties of the Fourier transform listed in the table to justify your answer.)

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega+4}{(j\omega+2)(j\omega+3)}$$

$$(j\omega+2)(j\omega+3)Y(j\omega) = (j\omega+4)X(j\omega)$$

$$((j\omega)^2 + 5j\omega + 6)Y(j\omega) = j\omega X(j\omega) + 4X(j\omega)$$

$$(j\omega)(j\omega)Y(j\omega) + 5j\omega Y(j\omega) + 6Y(j\omega) = j\omega X(j\omega) + 4X(j\omega)$$

by (16) (twice) by (16) by (16)

$$\boxed{\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = \frac{d}{dt}x(t) + 4x(t)}$$

C. T.   3

(10 pts) **5.** A CT signal $x(t)$ has Fourier transform

$$\mathcal{X}(\omega) = -2e^{(j-1)\omega}u(\omega+1).$$

Denote by $y(t)$ the signal obtained by delaying $x(t)$ by six seconds. Sketch a graph representing the magnitude $|\mathcal{Y}(\omega)|$ of the Fourier transform $\mathcal{Y}(\omega)$ of $y(t)$. (Justify your answer.)

by (10) with $t_0 = 6$

$$\mathcal{Y}(\omega) = -2e^{(j-1)\omega}e^{-j\omega 6}u(\omega+1) \checkmark$$

$$\cancel{y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} -2e^{(j-1)\omega}e^{-j\omega 6}u(\omega+1)\,d\omega}$$

$\mathcal{Y}(\omega) =$

$-2e^{j\omega-\omega-6j\omega}u(\omega+1) =$

$-2e^{-5j\omega-\omega}u(\omega+1)$

$|\mathcal{Y}(\omega)| =$

$|-2e^{-5j\omega}e^{-\omega}u(\omega+1)| =$

$-2|\cancel{e^{-5j\omega}}^{1}||\cancel{e^{-\omega}}^{1}||u(\omega+1)| =$

$-2u(\omega+1)$ $e^{-\omega}$ -4

-3