(22 pts) **1.** Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

	Yes	No
If $y(t) = x(2t)$, is the system causal?		X
If $y(t) = (t+2)x(t)$, is the system causal?		X
$y(1) = 3x(1)$		
If $y(t) = x(-t^2)$, is the system causal?		X
If $y(t) = x(t) + t - 1$, is the system memoryless?	X	
If $y(t) = x(t^2)$, is the system memoryless?		X
If $y(t) = x(t/3)$, is the system stable?	X	
If $y(t) = tx(t/3)$, is the system stable?		X
If $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, is the system stable?		X
If $y(t) = \sin(x(t))$, is the system time invariant?	X	
If $y(t) = u(t) * x(t)$, is the system LTI?	X	
If $y(t) = (tu(t)) * x(t)$, is the system linear?	X	

~~Quiz~~ on final

(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t+2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all $\sum$ signs disappear.)

$$y(t) = \int_{-\infty}^{\infty} x(t) * h(t)\, dt$$

$$= \int_{-\infty}^{\infty} x(\tau) * h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\; u(t-\tau+2)\, d\tau$$

but $u(\tau) = 0$

when $\tau \geq 0$

$$= \int_{0}^{\infty} e^{-\tau} u(t-\tau+2)\, d\tau$$

but $u(t-\tau+2) = 0$

when $t-\tau+2 \geq 0$

$t+2 \geq \tau \geq 0$

?

$$\int_{0}^{t+2} e^{-\tau}\, d\tau \Rightarrow -e^{-\tau}\Big|_{0}^{t+2}$$

$$= -e^{-(t+2)} + 1$$

$$y(t) = \left(1 - e^{-(t+2)}\right) u(t)$$

0

(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$.

$$x(t) = \frac{3e^{jt}}{1+j} \times \frac{(1-j)}{(1-j)} = \frac{3e^{jt} - 3je^{jt}}{\cancel{1-j+j+1}\ 2}$$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} \left| \frac{3e^{jt}}{1+j} \right|^2 dt = \int_{-\infty}^{\infty} \left| \frac{3}{1+j} \right|^2 dt \Rightarrow \int_{-\infty}^{\infty} \frac{9}{2j} \, dt$$

$$\int_{-\infty}^{\infty} -18j \, dt \Rightarrow -18jt \Big|_{-\infty}^{\infty} = -18j[\infty + \infty] = \infty$$

$$\boxed{E_\infty = \infty}$$

$$\cancel{P_\infty = \lim_{T \to 2T} \frac{E_\infty}{2T} = \boxed{1}}$$

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(15 pts) **4.** Compute the coefficients a_k of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}.$$

(Simplify your answer as much as possible.)

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\left(\frac{2\pi}{T}\right)t}\, dt$$

$$\sin(\pi t) = \frac{1}{2}e^{j\pi t} - \frac{1}{2j}e^{-j\pi t}$$

$$= \frac{1}{4}\int_0^2 \sin(\pi t)\, e^{-jk\left(\frac{\pi}{2}\right)t}\, dt + \cancel{\frac{1}{4}\int_2^4 0\, e^{-jk\left(\frac{\pi}{2}\right)t}\, dt}$$

$$= \frac{1}{4}\int_0^2 \sin(\pi t)\, e^{-jk\left(\frac{\pi}{2}\right)t}\, dt$$

$$= \frac{1}{4}\left[\frac{-\cos(\pi t)}{\pi} e^{-jk\left(\frac{\pi}{2}\right)t} + \frac{2\sin(\pi t)}{j\pi k} e^{-jk\left(\frac{\pi}{2}\right)t}\right|_0^2$$

$$\frac{\cos(2\pi)}{\pi} e^{-jk\pi} + \frac{2\sin 2t}{j\pi k} e^{-jk\pi t}$$

$$e^{-jk\pi} = (-1)^k$$

$$(-1)^k \pi \left(\frac{\cos 2\pi}{\pi} \mp \frac{2j\sin 2t}{k}\right) = a_k$$

$$a_0 = 0$$

5. A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

input		output
$x_0[n] = \delta[n]$	$\rightarrow$	$y_0[n] = \delta[n-1]$,
$x_1[n] = \delta[n-1]$	$\rightarrow$	$y_1[n] = 4\delta[n-2]$,
$x_2[n] = \delta[n-2]$	$\rightarrow$	$y_2[n] = 9\delta[n-3]$,
$x_3[n] = \delta[n-3]$	$\rightarrow$	$y_3[n] = 16\delta[n-4]$,
	$\vdots$	
$x_k[n] = \delta[n-k]$	$\rightarrow$	$y_k[n] = (k+1)^2\delta[n-(k+1)]$ for any integer k.

$y[n] =$

k

(10 pts) **a)** Can this system be time-invariant? Explain.

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$(k+1)^2\delta[n-(k-1)]$

$y[n] \rightarrow \boxed{\text{System}} \rightarrow \boxed{\text{TD}} \rightarrow \cancel{z[A] = \delta[n-k]}$

$Y[n] = (k+1)^2\,\delta[n-k-k+1]$

$\delta[n-k]$

$y[n] \rightarrow \boxed{\text{TD}} \rightarrow \boxed{\text{Sys}} \rightarrow z[A] = (k+1)^2\delta[n-k+1]$ → No, does not corespond

$Y[n] = (k+1)^2\,\delta[(n-k+1)-k]$

(10 pts) **b)** Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n-1]$?

0