Problem set 5

David Imberti

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Number 1 runs similarly to the construction of $\sup(AB) = \sup(A)\sup(B)$, and there is a talk on number 3 today. So, I really only see number 2 as being fortuitious to somewhat do:

2) Does $\sup E = \lim_{n\to\infty} \sup_{k\geq n} a_k$ where E is the set of all limit points of a_k?

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Lemma

$\forall\epsilon > 0, \exists N \ni \forall n \geq N a_n \leq \sup E + \epsilon$

Pf:

BWOC assume $\exists\epsilon > 0 \ni \forall n, a_n > \sup E + \epsilon$, but then this contradicts the definition of the $\sup E$ being the greatest upper bound of the set of limit points of a_n.

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Let $s_n = \sup_{k\geq n} a_k$, we show that $\lim_{\infty\to\infty} s_n = \sup E$, note that this is equivalent to what we need to show.

Furthermore, we know that s_n is monotonically decreasing by Bobby problem set 2.3.a, also by the lemma it is the great lower bound of s_n. Thus by a proof mutatis mutandis to 3.14 we have that $\lim_{n\to\infty} s_n = \sup E$.

Thus $\sup E = \lim_{n\to\infty} \sup_{k\geq n} a_k$, which is what we were trying to show.

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