ECE 544 Fall 2013
Problem Set 3
Due September 9, 2013

1. Read Chapters 3 and 4 of M. B. Pursley, *Introduction to Digital Communications* (MBP).

2. MBP Problems 3.9, 3.13, 4.4, and 4.9

3. Prove the *Theorem on Modulation:* $A(t)$, $B(t)$ jointly WSS. Θ r.v. uniform on $[0, 2\pi)$, statistically independent of $A(t)$, $B(t)$:

   Thm Part A: Then $X(t) = A(t)\cos(2\pi f_c t + \Theta)$ is WSS with $\mu_X = 0$ and

$$\begin{aligned} R_X(\tau) &= 0.5 R_A(\tau)\cos(2\pi f_c \tau) \\ &\updownarrow \\ S_X(f) &= 0.25[S_A(f - f_c) + S_A(f + f_c)] \end{aligned}$$

   Thm Part B: If $R_A(\tau) = R_B(\tau)$ and $R_{A,B}(\tau) = -R_{B,A}(\tau)$, then

$$X(t) = A(t)\cos(2\pi f_c t) - B(t)\sin(2\pi f_c t)$$

   has

$$\begin{aligned} R_X(\tau) &= R_A(\tau)\cos(2\pi f_c \tau) \\ &\qquad\qquad - R_{A,B}(\tau)\sin(2\pi f_c \tau) \\ &\updownarrow \\ S_X(f) &= 0.5[S_A(f - f_c) + S_A(f + f_c)] \\ &\quad + j0.5[S_{A,B}(f - f_c) - S_{A,B}(f + f_c)]. \end{aligned}$$

   Moreover, if $A(t)$, $B(t)$ are zero mean, then $X(t)$ has zero mean and is WSS.

   Thm Part C: Then

$$X(t) = A(t)\cos(2\pi f_c t + \Theta) - B(t)\sin(2\pi f_c t + \Theta)$$

   is zero mean, WSS with

$$\begin{aligned} R_X(\tau) &= 0.5[R_A(\tau) + R_B(\tau)]\cos(2\pi f_c \tau) \\ &\quad - 0.5[R_{A,B}(\tau) - R_{A,B}(-\tau)]\sin(2\pi f_c \tau) \\ &\updownarrow \\ S_X(f) &= 0.25[S_A(f - f_c) + S_B(f - f_c) \\ &\quad + S_A(f + f_c) + S_B(f + f_c)] \\ &\quad + j0.25[S_{A,B}(f - f_c) - S_{A,B}(f + f_c) \\ &\quad - S_{A,B}(-f + f_c) + S_{A,B}(-f - f_c)] \end{aligned}$$

4. Prove the maximum power transfer theorem for random process excitation. The circuit setup is standard – a voltage source $v_s(t)$ is in series with a source impedance $Z_s(f)$ to make a Thevenin equivalent circuit model. The source and the source impedance are considered fixed. Across the terminals of the Thevenin equivalent is a load impedance $Z_L(f)$. Assume that $v_s(t)$ is zero mean, WSS, with psd $S_{v_s}(f)$. To provide maximum power to the load (averaged over the ensemble) set

$$Z_L(f) = Z_s^*(f)$$

   for all frequencies f such that $S_{v_s}(f) > 0$. The resulting maximum power is

$$P_L^{max} = \int_{-\infty}^{\infty} \frac{S_{v_s}(f)}{4R_s(f)} \mathrm{d}f$$

   where $R_s(f) = \mathrm{Re}\{Z_s(f)\}$.