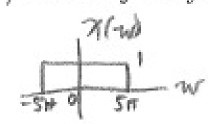
1. Consider the following CT signals:

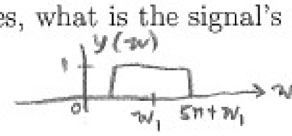
$$x(t) = \frac{\sin(5\pi t)}{\pi t},$$
$$y(t) = x(t)e^{j\omega_1 t},$$
$$z(t) = x(t)\cos(\omega_2 t). = x(t)\left(\tfrac{1}{2}e^{jw_2 t} + \tfrac{1}{2}e^{-jw_2 t}\right)$$

(5 pts) a) Is the signal $x(t)$ band limited? (Answer yes/no and justify.) If you answered yes, what is the signal's Nyquist rate?

yes, $X(w) \overset{\text{By 19}}{=} u(w+5\pi) - u(w-5\pi) =$ 

$\rightarrow w_m = 5\pi \rightarrow w_N = 2w_m = 10\pi$

(5 pts) b) Assuming that $\omega_1 > 0$, is the signal $y(t)$ band limited? (Answer yes/no and justify.) If you answered yes, what is the signal's Nyquist rate?

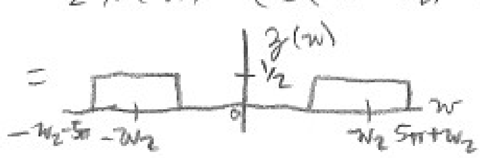
yes, $Y(w) \overset{\text{By 12}}{=} X(w - w_1)$ 

$\rightarrow w_m = 5\pi + w_1 \rightarrow w_N = 2w_m = 10\pi + 2w_1$

(5 pts) c) Assuming that $\omega_2 > 0$, is the signal $z(t)$ band limited? (Answer yes/no and justify.) If you answered yes, what is the signal's Nyquist rate?

yes, $Z(w) = \frac{1}{2\pi} X(w) * \left(2\pi\left(\tfrac{1}{2}\delta(w-w_2) + \tfrac{1}{2}\delta(w+w_2)\right)\right)$

$= \frac{1}{2} X(w) * \left(\delta(w-w_2) + \delta(w+w_2)\right)$

$=$ 

$\rightarrow w_m = 5\pi + w_2 \rightarrow w_N = 2w_m = 10\pi + 2w_2$

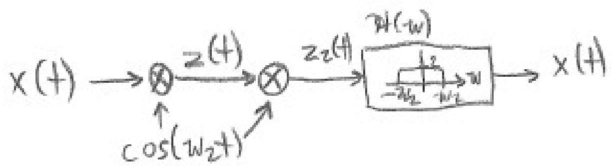
(10 pts) d) Can one recover x(t) from y(t)? (Answer yes/no. If you answer yes, explain how. If you answered no, explain why not.)

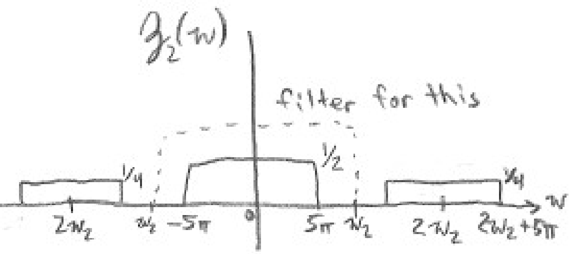
yes, multiply y(t) by $e^{-jw_1 t}$, By 12, this will shift $Y(w)$ back to $X(w)$.

$$\mathcal{F}\left(y(t)\,e^{-jw_1 t}\right) \overset{By\,12}{=} X(w - w_1 + w_1) = X(w)$$

(15 pts) e) Can one recover x(t) from z(t)? (Answer yes/no. If you answer yes, explain how. If you answered no, explain why not.)

yes, multiply z(t) by $\cos(w_2 t)$ and then low pass filter with a filter of gain 2 and $w_{co} = w_2$





(15 pts) f) Impulse-train sampling is used to obtain

$$x_p[n] = \sum_{k=-\infty}^{\infty} x(n)\delta(n - kN).$$

If the sampling period is $N = \frac{2}{11}$, will aliasing occur? Justify your answer.

$w_s = \frac{2\pi}{N} = \frac{2\pi}{2/11} = 2\pi \cdot \frac{11}{2} = 11\pi$

and $w_N = 10\pi$ (from a.)

$\Rightarrow$ $w_s > w_N$ $\rightarrow$ aliasing will not occur

(15 pts) g) Impulse-train sampling is used to obtain

$$y_p[n] = \sum_{k=-\infty}^{\infty} y(n)\delta(n - kN).$$

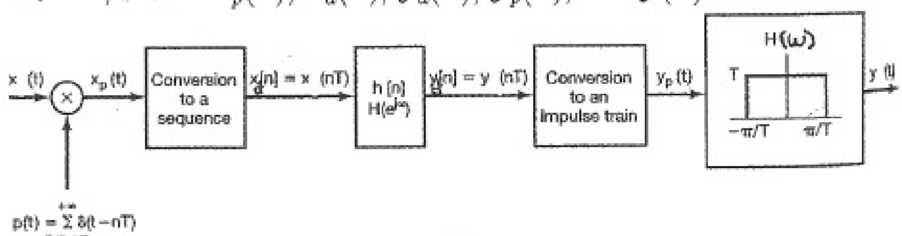
If $\omega_1 = -3\pi$ and the sampling period is $N = \frac{1}{6}$, will aliasing occur? Justify your answer.

$$w_s = \frac{2\pi}{N} = \frac{2\pi}{1/6} = 2\pi \cdot 6 = 12\pi$$

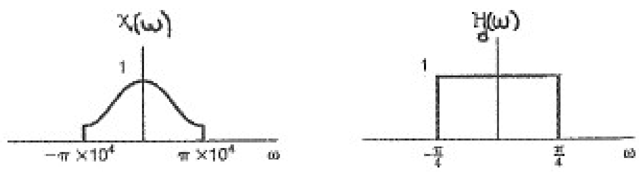
$w_N = 10\pi + 2w_1 = 10\pi - 6\pi = 4\pi$ (from b.)

$\rightarrow w_s > w_N \rightarrow$ aliasing will not occur

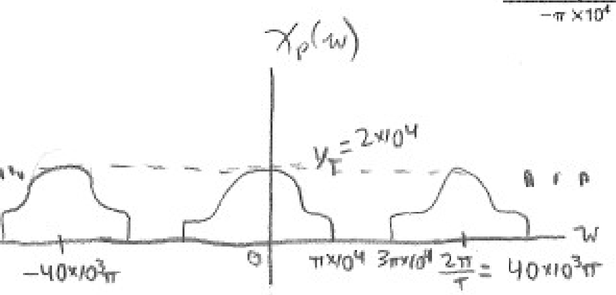
(20 pts) **2.** The following figure shows the overall system for filtering a CT signal using a DT filter. If $\mathcal{X}(\omega)$ and $\mathcal{H}_d(\omega)$ are as shown below, with $\frac{1}{T} = 20\text{kHZ}$, sketch $\mathcal{X}_p(\omega)$, $\mathcal{X}_d(\omega)$, $\mathcal{Y}_d(\omega)$, $\mathcal{Y}_p(\omega)$, and $\mathcal{Y}(\omega)$.

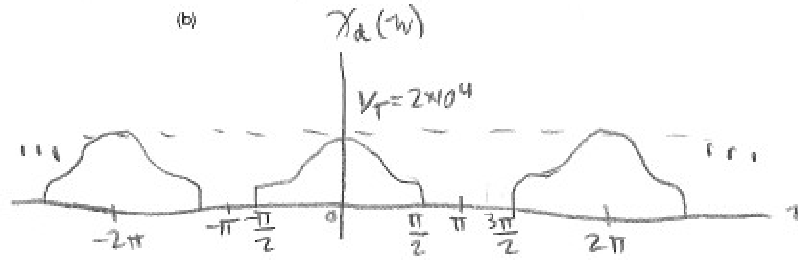


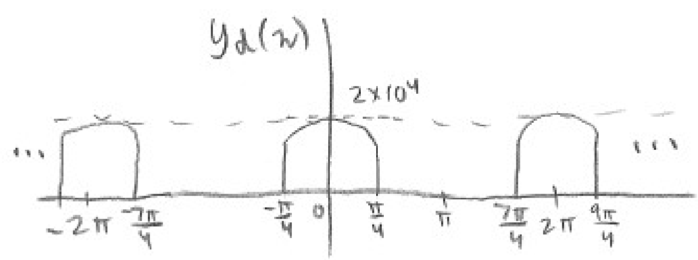
(a)

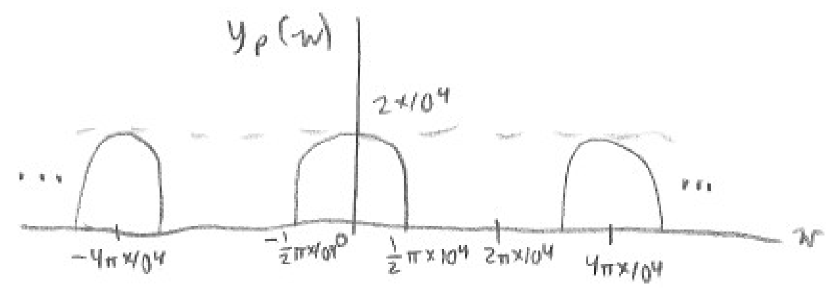


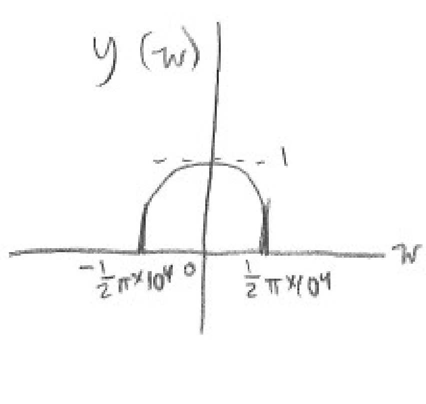
(b)











(15 pts) **3.** Using the definition of the Laplace transform (i.e. do not simply take the answer from the table), compute the Laplace transform of

$$x(t) = e^{-5t}u(-t)$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt = \int_{-\infty}^{\infty} e^{-5t} u(-t) e^{-st}\, dt$$

$$= \int_{-\infty}^{0} e^{-(5+s)t}\, dt$$

$$= \begin{cases} \text{divergent}, & s=-5 \\ \frac{-1}{5+s} e^{-(5+s)t}\Big]_{-\infty}^{0}, & \text{else} \end{cases} = \begin{cases} \text{divergent}, & s=-5 \\ \text{divergent}, & 5+s>0 = s>-5 \\ \frac{-1}{5+s} e^{-(5+s)t}\Big]_{-\infty}^{0}, & \text{else} \end{cases}$$

$$= \begin{cases} \text{divergent}, & s \geq -5 \\ \frac{-1}{5+s}, & \text{else} \end{cases}$$

$$= \frac{-1}{5+s}, \quad \text{ROC}: s < -5$$