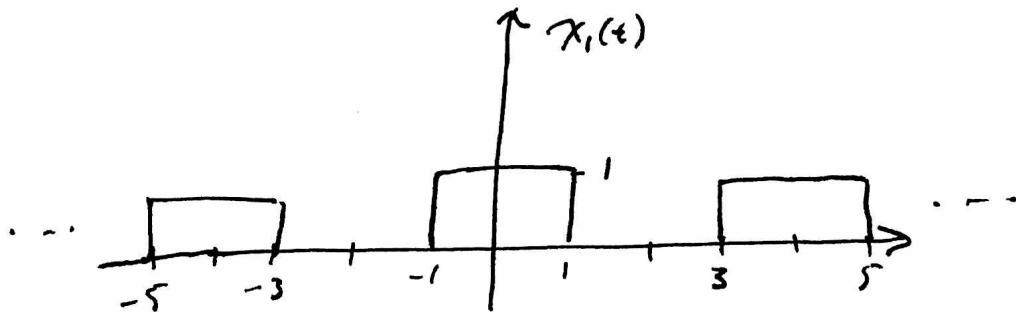


Ex



a) Find the Fourier series coefficients of  $x_i(t)$ .

$$a_k = \frac{1}{T_0} \int_{T_0} x_i(t) e^{-jk\omega_0 t} dt \quad T_0 = 4, \omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$$

$$= \frac{1}{4} \int_{-1}^1 e^{-jk\frac{\pi}{2}t} dt = \frac{1}{4} \cdot \frac{-1}{jk\frac{\pi}{2}} [e^{-jk\frac{\pi}{2}t}] \Big|_{-1}^1$$

$$= \frac{-1}{2jk\pi} [e^{-jk\frac{\pi}{2}} - e^{jk\frac{\pi}{2}}] = \frac{1}{2\pi} \sin(k\frac{\pi}{2})$$

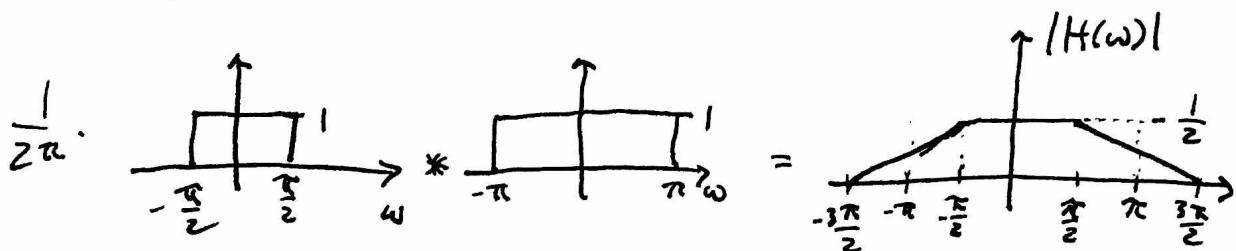
$$a_0 = \frac{1}{T_0} \int_{T_0} x_i(t) dt = \frac{1}{4} \int_{-1}^1 1 dt = \frac{1}{2}$$

b) Let  $h_i(t) = \frac{\sin(\frac{\pi}{2}t)}{\pi t}, \frac{\sin(\pi t)}{\pi t}$ . The system is

$$\xrightarrow{x_i(t)} \boxed{h_i(t)} \xrightarrow{y_i(t)} y_i(t) = x_i(t) * h_i(t)$$

→ Plot  $|H(\omega)|$

$$h_i(t) = g(t) \cdot f(t) \longleftrightarrow \frac{1}{2\pi} (G(\omega) * F(\omega))$$



$\rightarrow$  Find  $y_1(t)$ .

If  $y_1(t) \xleftrightarrow{\text{FS}} b_R$ , then  $b_R = a_R H_i(\omega_0 R)$ .

$$b_0 = a_0 \cdot H(0) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$b_1 = b_{-1} = a_1 H_i(\omega_0 \cdot 1) = a_1 H_i\left(\frac{\pi}{2}\right) = \frac{1}{2\pi} \sin\left(\frac{\pi}{2}\right) \cdot \frac{1}{2}$$

$$= \frac{1}{2\pi}$$

$$b_2 = b_{-2} = a_2 H_i(\pi) = \frac{1}{2\pi} \sin\left(\pi\right) \cdot \frac{1}{4}$$

$$= 0$$

$b_R = 0$  for all  $R > 2$  because  
 $H_i(\omega)$  filters out those  
frequencies.

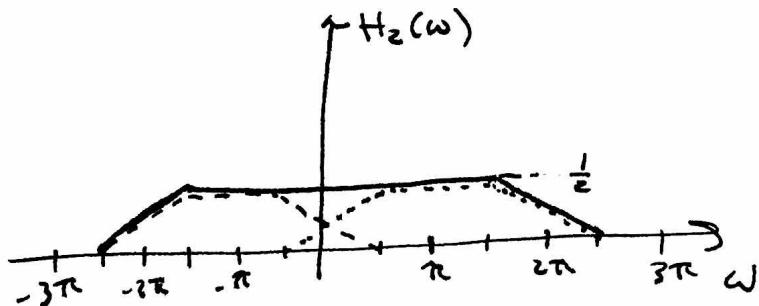
\*  $b_1 = b_{-1}$  because  $a_R = a_{-R}$   
and  $H_i(\omega) = H_i(-\omega)$ . If either  
 $a_R$  or  $H_i(\omega)$  were not symmetric,  
you would calculate each  
separately.

$$y_1(t) = \sum_{k=-\infty}^{\infty} b_k e^{j k \frac{\pi}{2} t} = \frac{1}{2\pi} e^{-j \frac{\pi}{2} t} + \frac{1}{4} + \frac{1}{2\pi} e^{j \frac{\pi}{2} t}$$

$$= \frac{1}{4} + \frac{1}{\pi} \cos\left(\frac{\pi}{2}t\right)$$

c) Let  $h_2 = h_1(t) \cos(\pi t)$ . Find  $y_2(t) = x_1(t) * h_2(t)$

$$H_2(\omega) = \frac{1}{2} (H(\omega - \pi) + H(\omega + \pi))$$



If  $y_2(t) \xrightarrow{\text{Fs}} c_k$ , then  $c_k = a_k H(\omega_0 k)$

$$c_0 = a_0 H(0) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$c_1 = c_{-1} = a_1 H\left(\frac{\pi}{2}\right) = \frac{1}{\pi} \sin\left(\frac{\pi}{2}\right) H\left(\frac{\pi}{2}\right) = \frac{1}{2\pi}$$

$$c_2 = c_{-2} = a_2 H_2(\pi) = \frac{1}{\pi} \sin\left(\frac{\pi}{2}\right) H(\pi) = 0$$

$$c_3 = c_{-3} = a_3 H_2\left(\frac{3\pi}{2}\right) = \frac{1}{3\pi} \sin\left(\frac{3\pi}{2}\right) H\left(\frac{3\pi}{2}\right) = -\frac{1}{6\pi}$$

$$c_4 = c_{-4} = a_4 H(2\pi) = \frac{1}{4\pi} \sin(2\pi) H(2\pi) = 0$$

$$c_k = 0 \text{ for } k > 4$$

$$\begin{aligned} y_2(t) &= -\frac{1}{6\pi} e^{-j\frac{3\pi}{2}t} + \frac{1}{2\pi} e^{-j\frac{\pi}{2}t} + \frac{1}{4} + \frac{1}{2\pi} e^{j\frac{3\pi}{2}t} - \frac{1}{6\pi} e^{j\frac{3\pi}{2}t} \\ &= \frac{1}{4} + \frac{1}{\pi} \cos\left(\frac{\pi}{2}t\right) - \frac{1}{3\pi} \cos\left(\frac{3\pi}{2}t\right) \end{aligned}$$

d) Make a highpass filter

$$H_{hp}(\omega) = \begin{cases} 1, & |\omega| > 20\pi \\ 0, & \text{else} \end{cases}$$

What values of  $a_k$  will be set to zero by this filter for the input  $x_1(t)$ ?

$\rightarrow a_k$  has the period  $\frac{\pi}{2}$ , so  $20\pi$  would be the ~~corner~~ corresponds to  $k\omega_0 = 20\pi \Rightarrow k = \frac{20\pi}{\frac{\pi}{2}} = 40$

Then  $H_{hp}(\omega)$  will zero (filter out)  $a_k$  where  $|k| \leq 40$ .

e) How do your answers change if  $x_2(t) = k_1(2t)$ ?

Your Fourier series coefficients are unchanged.

$$x_2(t) \xrightarrow{\text{FS}} a_k = \frac{1}{\pi} \sin\left(\frac{\pi}{2}k\right) \text{ for } k \neq 0$$

$$a_0 = \frac{1}{2}$$

When filtering, remember that your new  $\omega_0$  is  $\pi$  after applying the time scale.

$$\rightarrow k_1(t) * h_1(t) \longleftrightarrow d_k = a_k H_1(\pi k)$$

$$d_0 = a_0 H_1(0) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$d_1 = d_{-1} = a_1 H_1(\pi) = \frac{1}{\pi} \sin\left(\frac{\pi}{2}\right) \frac{1}{4} = \frac{1}{4\pi}$$

$$d_k = 0 \text{ for } k \geq 2$$

$$x_1(t) * h_1(t) = \sum_{k=-\infty}^{\infty} d_k e^{j\pi k t}$$

$$= \frac{1}{4\pi} e^{-j\pi t} + \frac{1}{4} + \frac{1}{4\pi} e^{j\pi t}$$

$$= \frac{1}{4} + \frac{1}{2\pi} \cos(\pi t)$$

$$\rightarrow k_2(t) * h_2(t) \longleftrightarrow e_k = a_k H_2(\pi k)$$

I probably shouldn't  
use  $e$  as a  
variable like this,

$$e_0 = a_0 H_2(0) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$e_1 = e_{-1} = a_1 H_2(\pi) = \frac{1}{\pi} \sin\left(\frac{\pi}{2}\right) \cdot \frac{1}{2} = \frac{1}{2\pi}$$

$$e_2 = e_{-2} = a_2 H_2(2\pi) = \frac{1}{2\pi} \sin(\pi) \cdot \frac{1}{4} = 0$$

$$e_k = 0 \text{ for } k \geq 3$$

$$x_2(t) * h_2(t) = \sum_{k=-\infty}^{\infty} e_k e^{j\pi k t} = \frac{1}{2\pi} e^{-j\pi t} + \frac{1}{4} + \frac{1}{2\pi} e^{j\pi t}$$

$$= \frac{1}{4} + \frac{1}{\pi} \cos(\pi t)$$

$\rightarrow$  Lastly, for the highpass filter, the component at  $k \cdot \pi = 20\pi \Rightarrow k = 20$  is the cutoff.

$$\Rightarrow x_2(t) \xrightarrow{\text{FS}} a_k$$

$$a_k = 0 \text{ for } |k| \leq 20 \text{ after filtering.}$$