

# ECE 440L

## Experiment 2: Linear and Non-Linear Systems (1 week)

### I. OBJECTIVES

Upon completion of this experiment, you should be able to:

1. Characterize linear, time-invariant systems in the time and frequency domains using signal generators and oscilloscopes.
2. Build a simple scalar network analyzer to measure the frequency response of a low frequency LTI system.
3. Measure the inter-modulation distortion of a simple non-linear system.

### II. INTRODUCTION

Frequently you will need to characterize linear systems in terms of their time-domain responses (impulse response, step response) or their frequency domain response. In addition, most ‘linear’ systems are not perfectly linear; they exhibit weak non-linearities. These non-linearities can have a profound impact on communication systems and it is important to study and characterize them in terms of the ‘inter-modulation’ products.

### III. PRELAB

1. Define impulse response and frequency response for linear time-invariant (LTI) systems.
2. The frequency response of an LTI system is given by  $|H(f)|e^{j2\pi\theta(f)}$ .  
The input to this system is given by  $x(t) = 2\cos(2\pi 3000t) + 3\cos(2\pi 4700t)$ .  
What is the output?
3. Consider a non-linear system defined by  $y(t) = x(t) + ax(t)^2 + bx(t)^3$ .  
Here,  $a$  and  $b$  are constants,  $x(t)$  is the input to the system, and  $y(t)$  is the output.  
If  $x(t)$  is given by  $x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$ , determine  $y(t)$  and its spectrum,  $Y(f)$ . Also determine  $X(f)$ , the spectrum of  $x(t)$ .  
(Refer to [1, Appendix G.2, G.3] for a useful list of formulae.  
Or use this hint: The **MATLAB** command *simple* can be used to simplify expressions.)
4. When a LTI system is driven by a pure sinusoidal input, the output is also a pure sinusoid at the same frequency. Is this true for linear time-variant systems? Justify.  
Is this true for non-linear systems? Justify.

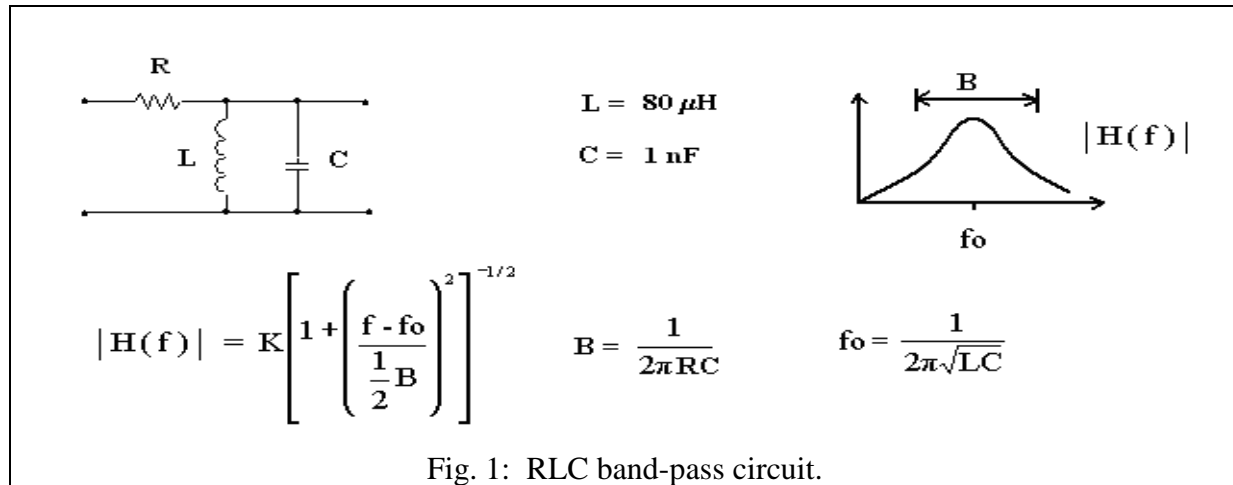
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## IV. EXPERIMENT

The goals of this experiment are to characterize an LTI system in the time and frequency domains and to characterize a weakly non-linear system in terms of its inter-modulation distortion (IMD).

### 1. Measuring the Frequency Response of an LTI System.

1 a. Build the RLC band-pass filter shown in Figure 1.



Select C for  $f_0 = 400\text{-}500$  kHz. Select R so that the 3-dB bandwidth, B, is 100 kHz. Determine the step response of this circuit (use a square wave of appropriate frequency as input and observe the output on an **oscilloscope**).

1 b. Manually plot the frequency response of this band-pass filter from 0 to 800 kHz. Generate sinusoids at frequencies in this range and use them as inputs to the BPF. Observe the output of the BPF on an **oscilloscope**. Determine the maximum gain, and the 3-dB cut-off frequencies. Record  $V_{in}(f)$  and  $V_{out}(f)$  and  $|H(f)| = |V_{out}(f) / V_{in}(f)|$ . Focus on  $|H(f)|$ , the amplitude of the frequency response. Ignore the phase function.

1 c. Build a simple network analyzer.

Automatic Frequency Scanning.

In 1 b, above, the input frequency to the filter was manually scanned. Frequency scanning may be automated by using a voltage-controlled oscillator (VCO). The output frequency of a VCO depends on its input voltage.

You will be using the 3314A as a VCO. The characteristics of the 3314A VCO are:

The frequency changes roughly 10% of the displayed nominal frequency per volt of input.

The input voltage must lie between -10 volts and +1 volt.

For example, if the displayed frequency is 50 kHz, and the input is +0.5 volts, the output frequency is 52.5 kHz ( $50 + 0.5 \cdot 10\% \cdot 50$ ).

- 1 c. Build a low frequency network analyzer. (Continued.)

The frequency range must be scanned very slowly to get sensible results.

A rule of thumb is:

The product of sweep-time and frequency span must be much larger than 1.

A signal that satisfies this rule is a wide-band frequency-modulated signal.

Generate a symmetric triangular wave (with as much DC offset as required) using the 33220A and use it as input ( between -10 and +1 volts) to the 3314A VCO .

Use the VCO output as input to the BPF. Connect the triangular wave as X-input to the oscilloscope, the output of the BPF as Y-input and put the scope in X-Y mode.

Measure and record the bandwidth and maximum gain of the BPF of 1b using your simple low-frequency **scalar network analyzer**.<sup>1</sup> Compare to the results of 1b.

## 2. Measuring the Inter-modulation Distortion of a “Linear” system.

Many ‘linear’ systems show small amounts of non-linearity especially when driven by large input signals. This is true of the low-noise amplifiers, power amplifiers and filters commonly used in communication systems. Non-linearity results in unwanted frequency components like inter-modulation products and harmonics. For example, the specification of an RF amplifier will include the third-order intercept point specifying the extent of the third-order inter-modulation product.

- 2 a. Why isn’t the RLC circuit, in 1 above, chosen to be tested with the two-tone test?

- 2 b. The two-tone test.

Perform the two tone test on the **455 kHz BPF** at your station. This filter-amplifier combination has an 18 kHz bandwidth centered at 455 kHz. This unit has “weak non-linearities” that produce measurable inter-modulation distortion for a 4 volt peak-to-peak, two-tone input signal.

The two-tone test is performed as follows:

Generate two sinusoids of **equal** amplitude using the two function generators.

Pick their frequencies as 450 kHz and 460 kHz. Use 1 volt amplitudes to begin.

Add these sinusoids and use the resultant “two-tone” signal as input to the BPF.

Analyze the BPF output signal using a logarithmic scale of the spectrum analyzer.

**NOTE:** The spectrum analyzer cannot handle large input voltages..

- Always use a **40 dB attenuator** between the signal and the SA input.

- The **40 dB attenuator** is just a 4950Ω resistor.

- This resistor and the 50Ω input resistance of the SA form a 100:1 voltage divider.

If at any point, the SA displays a ‘**Caution: Overload on Input**’ sign, disconnect the signal at the SA input and reduce all amplitudes.

<sup>1</sup> The 4395A can be used as a **vector network analyzer**; “vector” refers to the fact that it is capable of providing both phase and amplitude information. The proper use of the network analyzer requires an understanding of calibration procedures. We will not be doing this in the lab. You can read more about network analyzers in [2], [3].

2 b. The two-tone test. (Continued.)

Record the amplitudes of the:

fundamentals:  $f_1 = 450 \text{ kHz}$  and  $f_2 = 460 \text{ kHz}$

second harmonics :  $2f_1 = 900 \text{ kHz}$  and  $2f_2 = 920 \text{ kHz}$

third harmonics:  $3f_1 = 1350 \text{ kHz}$  and  $3f_2 = 1380 \text{ kHz}$

inter-modulation terms

second-order  $|f_1 \pm f_2|$ : 10 kHz and 910 kHz

third order  $|2f_1 \pm f_2|, |f_1 \pm 2f_2|$ : 440 kHz, 470 kHz, 1360 kHz, 1370 kHz

2 c. Effect of reduced amplitude on the 3<sup>rd</sup> order **in-band** inter-modulation distortion.

To reduce the amplitude of the input signal while keeping the two tones equal in power, insert the HP350D Attenuator (see Appendix) between the adder and the BPF.

Observe the effect of reducing the input amplitude.

Gather data spaced over **two decades** of input magnitude.

Measure the fundamentals (450 kHz and 460 kHz) and the in-band third order inter-modulation terms (440 kHz and 470 kHz).

Plot the RMS voltages of the in-band third order terms vs. the amplitude of either fundamental. **Use a logarithmic scale for both axes.**

Discuss your results with respect to Prelab Question 3.

2 d. Why are the **in-band** third-order inter-modulation terms especially important?

## V. REPORT

Document all the readings you have obtained and any conclusions you draw in your report. Attach a copy of your lab record to the report. Answer any specific questions asked in the lab manual.

## VI. APPENDIX

### HP350D Attenuator

The HP350D is a passive resistive-bridge attenuator. For the HP350D attenuator to work roughly as calibrated, it is necessary that the input and output resistances be  $600\Omega$ . For example, if the input is driven directly by a  $50\Omega$  signal generator like the 33220A and the output resistance is  $50\Omega$ , the attenuation will not be as marked. At the very least, one of the two terminations must be  $600\Omega$ .

### Network Analyzers

A real linear system is not characterized by one frequency (or time) response function. The linear systems you will encounter in the lab are ‘two-ports’; four response functions (time or frequency) are required to characterize ‘two-port’ linear systems. There are many equivalent formulations for these four function parameters. These include the Z-parameters, G-parameters, H-parameters, Y-parameters, S-parameters, etc. The S-parameters are the most commonly used, especially at higher frequencies. They are the only ones that can be measured at high frequencies. A network analyzer measures one or more of these four S-parameters as functions of frequency. (Typically, two (transmission/reflection) at a time or all four at a time.)

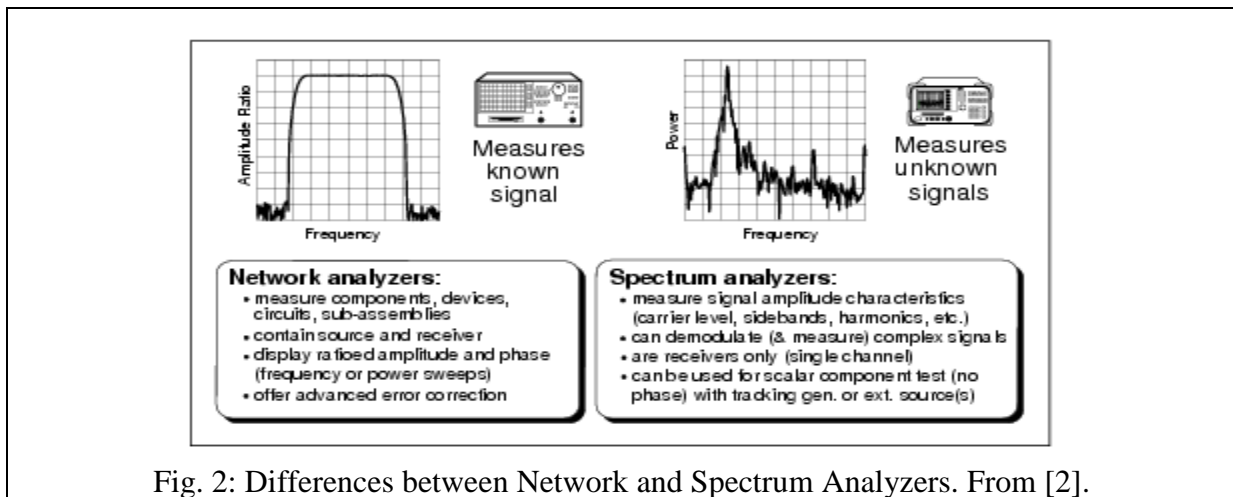


Fig. 2: Differences between Network and Spectrum Analyzers. From [2].

Roughly speaking, in addition to the forward gain, you also need to know the input resistance, the output resistance, and the reverse gain to characterize a two-port. And all of these can be functions of frequency. Vector network analyzers determine both the amplitude and the phase associated with these functions of frequency. Scalar network analyzers determine only the amplitude.

## REFERENCES

- [1] R. E. Ziemer and W. H. Tranter, *Principles of Communications*, 5th ed. Hoboken, NJ: John Wiley, 2002.
- [2] Agilent Technologies, “Understanding the Fundamental Principles of Vector Network Analysis,” Agilent Technologies, AN 1287-1, 2000. <http://cp.literature.agilent.com>.
- [3] —, “Exploring the Architectures of Network Analyzers,” Agilent Technologies, AN 1287-2, 2000. <http://cp.literature.agilent.com>

