

1. Use the system equation to show the following properties. Show all of your work when proving/disproving properties.

$$y[n] = \begin{cases} 0 & n < 2 \\ \alpha^{-x[n]} & n \geq 2 \end{cases}$$

- (a) (5 points) Linearity
- (b) (5 points) Time invariance
- (c) (5 points) Memory
- (d) (5 points) Causality
- (e) (5 points) BIBO stability

a) $y[n] = \alpha^{-x[n]} u[n-2]$

$$\begin{aligned} S\{ax_1[n] + bx_2[n]\} &= \alpha^{-(ax_1[n] + bx_2[n])} u[n-2] \\ &= \alpha^{-ax_1[n]} \alpha^{-bx_2[n]} u[n-2] \\ &\neq aS\{x_1[n]\} + bS\{x_2[n]\} \\ &= a \cdot \alpha^{-x_1[n]} u[n-2] + b \alpha^{-x_2[n]} u[n-2] \end{aligned}$$

Nonlinear

b) $y[n-n_0] = \alpha^{-x[n-n_0]} u[n-n_0-2]$

$$x_d = \kappa [n - n_0]$$

$$\begin{aligned} y_d[n] &= S\{x_d[n]\} = \alpha^{-\kappa a[n]} u[n-2] \\ &= \alpha^{-\kappa [n-n_0]} u[n-2] \neq y[n-n_0] \end{aligned}$$

Time varying

c) $y[n]$ depends on only the current value of the input.
Memoryless

d) $y[n]$ does not depend on the future values of $x(n)$, only
the current value.
Causal

e) If $|\alpha| < 1$, where $\alpha \neq 0$

for $|x[n]| < B$

$\alpha^B < y[n] < \alpha^{-B}$ (an α with magnitude less than one raised to a large positive becomes very small, raised to a large negative becomes large)

If $|\alpha| > 1$

$\alpha^{-B} < y[n] < \alpha^B$

If $\alpha = 0$

$y[n] = 0$

BIBO for any α .

2. Use the system equation to show the following properties. Show all of your work when proving/disproving properties.

$$y(t) = \tan(x(t+1))$$

- (a) (5 points) Linearity
- (b) (5 points) Time invariance
- (c) (5 points) Memory
- (d) (5 points) Causality
- (e) (5 points) BIBO stability

a) $\{ \sum a x_1(\epsilon) + b x_2(\epsilon) \} = \tan(a x_1(\epsilon+1) + b x_2(\epsilon+1))$

$$\neq a \{ x_1(\epsilon) \} + b \{ x_2(\epsilon) \}$$

$$= a \tan(x_1(\epsilon+1)) + b \tan(x_2(\epsilon+1))$$

Nonlinear

b) $y(t-t_0) = \tan(x(t-t_0+1))$

let $x_d(\epsilon) = x(\epsilon-t_0)$, then

$$y_d(\epsilon) = \{ x_d(\epsilon) \} = \tan(x_d(\epsilon+1))$$

$$= \tan(x(\epsilon+1-t_0)) \neq y(t-t_0)$$

Time invariant

- c) The current value depends on a future value, for example
 $y(0) = \tan(x(1))$.

Memory

- d) As above, $y(0) = \tan(x(1))$, so it depends on the future and is not causal.

Anticausal

- e) The tangent has discontinuities where it goes to infinity, eg $\tan\left(\frac{\pi}{2}\right) = \infty$

Therefore, a bounded input can lead to an unbounded output.

Unstable

3. Use the following system equation to answer the questions below.

$$y(t) = \int_{-\infty}^t (x(\tau + 1) - x(\tau - 2)) d\tau$$

(a) (8 points) Calculate the impulse response, $h(t)$.

(b) (3 points) Plot $h(t)$.

If you were not able to find $h(t)$, use $h(t) = e^{-2t}u(t)$ for the rest of the question.

Do not give two answers! Only use if you do not have an answer to part a.

(c) (3 points) Is the system causal? Show all of your work.

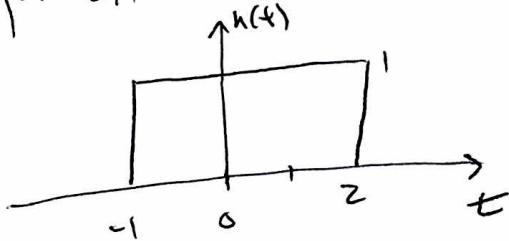
(d) (3 points) Is the system BIBO stable? Show all of your work.

(e) (8 points) Calculate $y(t) = x(t) * h(t)$ for $x(t) = \begin{cases} e^{-t} & 0 < t < 2 \\ 0 & \text{else} \end{cases}$

a) If $x(t) = \delta(t)$, then $y(t) = h(t)$

$$\begin{aligned} h(t) &= \int_{-\infty}^t (\delta(\tau + 1) - \delta(\tau - 2)) d\tau \\ &= u(t+1) - u(t-2) \end{aligned}$$

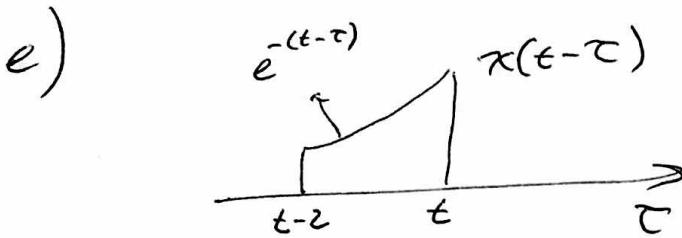
b) *Answers for alternate $h(t)$ in back.



c) $h(t) \neq 0$ for $t < 0$, for example $h(-0.5) = 1$
 \Rightarrow Not causal

d) $\int_{-\infty}^0 |h(t)| dt = \int_0^2 dt = 3 < \infty$

BIBO stable



1) if $t < -1$, no overlap

$$y(t) = 0$$

2) $t > -1$, $t-2 < -1$ $-1 < t < 1$, partial overlap

$$\begin{aligned} y(t) &= \int_{-1}^t e^{-(t-\tau)} d\tau = e^{-t} \cdot [e^\tau]_{-1}^t = e^{-t} (e^t - e^{-1}) \\ &= 1 - e^{-(t+1)} \end{aligned}$$

3) $t > 1$, $t-2 > -1$ $1 < t < 2$, full overlap

$$\begin{aligned} y(t) &= \int_{t-2}^t e^{-(t-\tau)} d\tau = e^{-t} [e^\tau]_{t-2}^t \\ &= e^{-t} [e^t - e^{t-2}] = 1 - e^{-2} \end{aligned}$$

4) $t-2 < 2$, $t > 2$ $2 < t < 4$, partial overlap

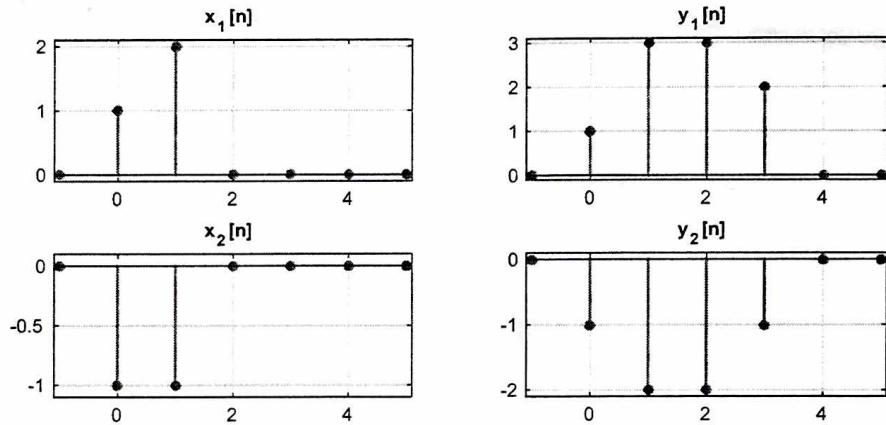
$$\begin{aligned} y(t) &= \int_{t-2}^2 e^{-(t-\tau)} d\tau = e^{-t} [e^\tau]_{t-2}^2 = e^{-t} (e^2 - e^{t-2}) \\ &= e^{2-t} - e^{-2} \end{aligned}$$

5) $t-2 > 2$, $t > 4$, no overlap $y(t) = 0$

$$y(t) = \begin{cases} 0 & t < -1 \\ 1 - e^{-(t+1)} & -1 < t < 1 \\ 1 - e^{-2} & 1 < t < 2 \\ e^{2-t} - e^{-2} & 2 < t < 4 \\ 0 & t > 4 \end{cases}$$

4.

The signals $x_1[n]$ and $x_2[n]$ pass through a discrete LTI system, giving the outputs $y_1[n]$ and $y_2[n]$.



(a) (7 points) Calculate and plot the impulse response, $h[n]$, of this system.

(b) (5 points) Is this system causal? Show all of your work.

If you did not find $h[n]$, use $h[n] = y_1[n]$ for the rest of this question. Do not give two answers! Only use if you do not have an answer to part a.

(c) (5 points) Is this system BIBO stable? Show all of your work.

(d) (8 points) Calculate and plot the output if the input is $x[n] = n(u[n] - u[n-4])$.

$$a) \quad x_1[n] + x_2[n] = \delta[n-1] \Rightarrow y_1[n] + y_2[n] = h[n-1]$$

$$h[n-1] = \sum_{n=0}^{\infty} \{1, -1, 3, -2, 3, -2, 2, -1\} = \sum_{n=0}^{\infty} \{1, 1, 1, 1, 1, 1, 1, 1\}$$

$$h[n] = \sum_{n=0}^{\infty} \{1, 1, 1\}$$

$h[n]$

$$b) \quad h[n] = 0 \text{ for all } n < 0 \Rightarrow \boxed{\text{Causal}}$$

* Answers for alternate in back.

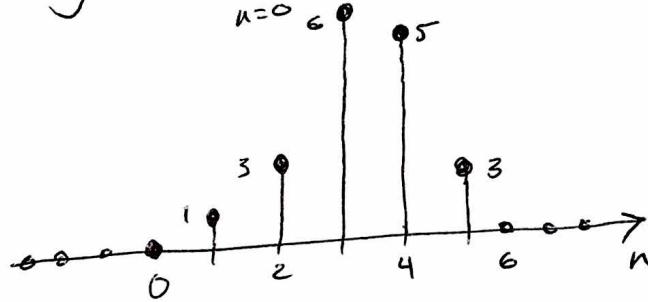
$$c) \sum_n |h[n]| = 1+1+1 = 3 < \infty \quad \boxed{\text{BIBO stable}}$$

$$d) x[n] = \sum_{n=0}^{\infty} f^n$$

Using superposition

$$\sum_{n=0}^{\infty} f^n * \sum_{n=0}^{\infty} f^n = \frac{0, \quad 1, \quad 2, \quad 3}{0, \quad 1, \quad 2, \quad 3} \\ \underline{0, \quad 1, \quad 3, \quad 6, \quad 5, \quad 3}$$

$$y[n] = \sum_{n=0}^{\infty} f^n$$



3. Use the following system equation to answer the questions below.

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(a) (8 points) Calculate the impulse response, $h(t)$.

(b) (3 points) Plot $h(t)$.

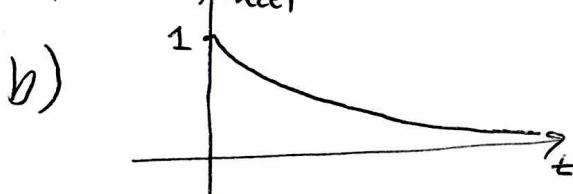
If you were not able to find $h(t)$, use $h(t) = e^{-2t}u(t)$ for the rest of the question.
Do not give two answers! Only use if you do not have an answer to part a.

(c) (3 points) Is the system causal? Show all of your work.

(d) (3 points) Is the system BIBO stable? Show all of your work.

(e) (8 points) Calculate $y(t) = x(t) * h(t)$ for $x(t) = \begin{cases} e^{-t} & 0 < t < 2 \\ 0 & \text{else} \end{cases}$

Alternate $h(t)$.

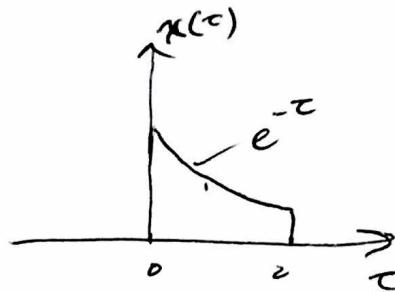
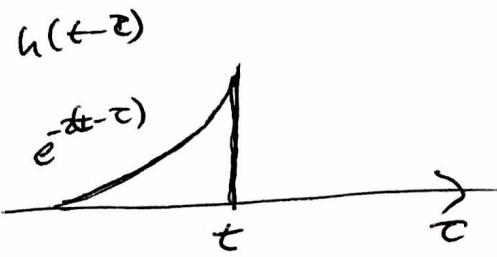


b) $h(t) = e^{-t}$ for $t > 0$ \Rightarrow causal

c) $h(t) = 0$ for all $t < 0$ \Rightarrow causal

d) $\int_{-\infty}^{\infty} |h(t)| dt = \int_0^{\infty} e^{-t} dt = -[e^{-t}]_0^{\infty} = -[0 - 1] = 1 < \infty \Rightarrow$ stable

e)



1) $t < 0 \Rightarrow$ no overlap $y(t) = 0$

2) $t > 0, t < 2 \quad 0 < t < 2 \quad$ partial overlap

$$y(t) = \int_0^t e^{-2\tau} e^{2\tau} e^{-\tau} d\tau = e^{-2t} \int_0^t e^\tau d\tau$$

$$= e^{-2t} [e^\tau]_0^t = e^{-2t} [e^t - 1] = e^{-t} - e^{-2t}$$

3) $t > 2 \quad$ full overlap

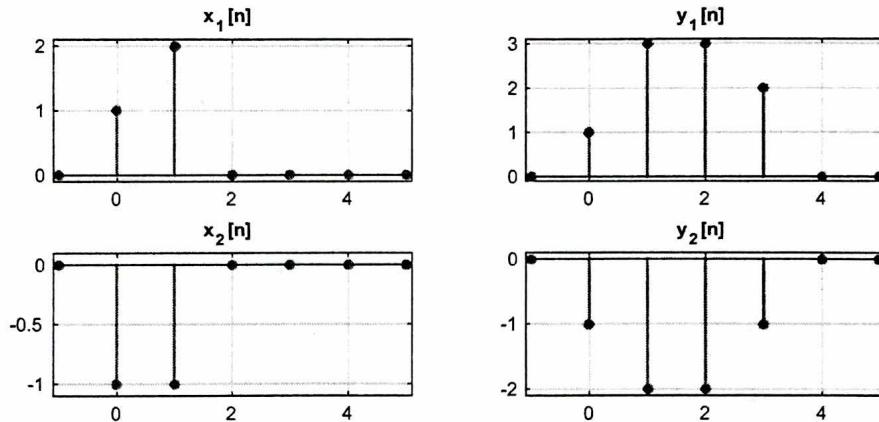
$$y(t) = \int_0^2 e^{-2\tau} e^{2\tau} e^{-\tau} d\tau = e^{-2t} [e^\tau]_0^2$$

$$= e^{-2t} [e^2 - 1] = e^{-2(t-1)} - e^{-2t}$$

$$y(t) = \begin{cases} 0 & t < 0 \\ e^{-t} - e^{-2t} & 0 < t < 2 \\ e^{-2(t-1)} - e^{-2t} & t > 2 \end{cases}$$

4.

The signals $x_1[n]$ and $x_2[n]$ pass through a discrete LTI system, giving the outputs $y_1[n]$ and $y_2[n]$.



- (a) (7 points) Calculate and plot the impulse response, $h[n]$, of this system.
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If you did not find $h[n]$, use $h[n] = y_1[n]$ for the rest of this question. Do not give two answers! Only use if you do not have an answer to part a.
- (c) (5 points) Is this system BIBO stable? Show all of your work.
- (d) (8 points) Calculate and plot the output if the input is $x[n] = n(u[n] - u[n - 4])$.

Alternate $h[n] = y_1[n]$

b) $h[n] = 0$ for $n < 0 \Rightarrow$ causal

c) $\sum_n |h[n]| = 1 + 3 + 3 + 2 = 9 < \infty \Rightarrow$ stable

$$d) \quad x[n] = \sum_{n=0}^{\infty} \{0, 1, 2, 3\}$$

using superposition

$$\{1, 3, 3, 2\} * \{0, 1, 2, 3\} =$$

$$\begin{array}{cccccc} 0, & 1, & 2, & 3 \\ 0, & 3, & 6, & 9 \\ 0, & 3, & 6, & 9 \\ 0, & 2, & 4, & 6 \end{array}$$

$$\overline{0, \ 1, \ 5, \ 12, \ 17, \ 13, \ 6}$$

$$\sum_{n=0}^{\infty}$$

$$y[n] = \sum_{n=0}^{\infty} \{0, 1, 5, 12, 17, 13, 6\}$$

