CSFT and Polar Coordinates

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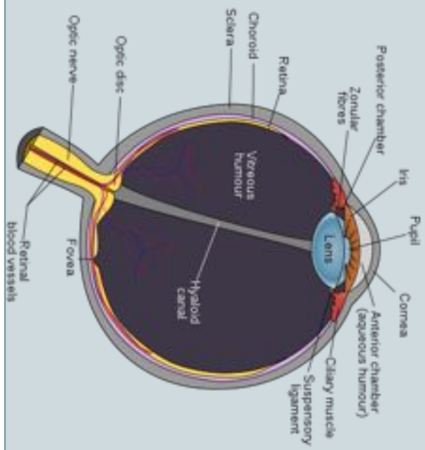
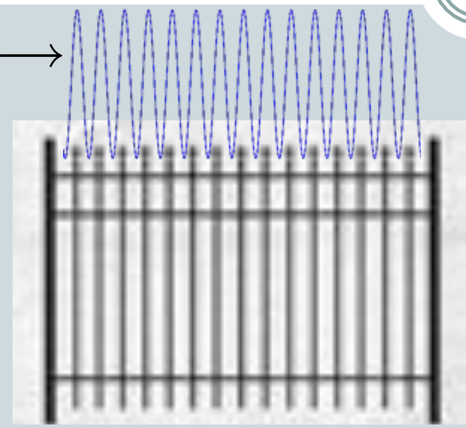
MONDAY, NOVEMBER 11TH, 2009

HECTOR SANTOS-VILLALOBOS

The fence analogy

$\cos(w_1 x) \longrightarrow$

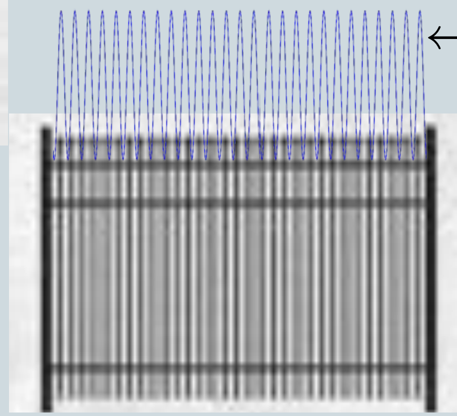




Near

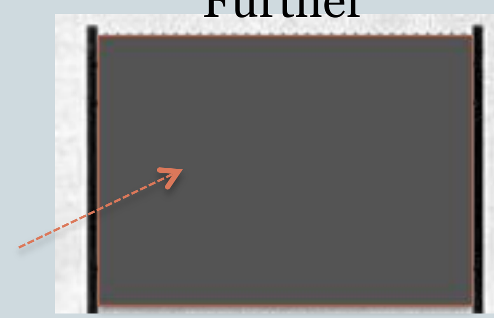
$w_1 << w_2$

$\longleftarrow \cos(w_2 x)$



Far

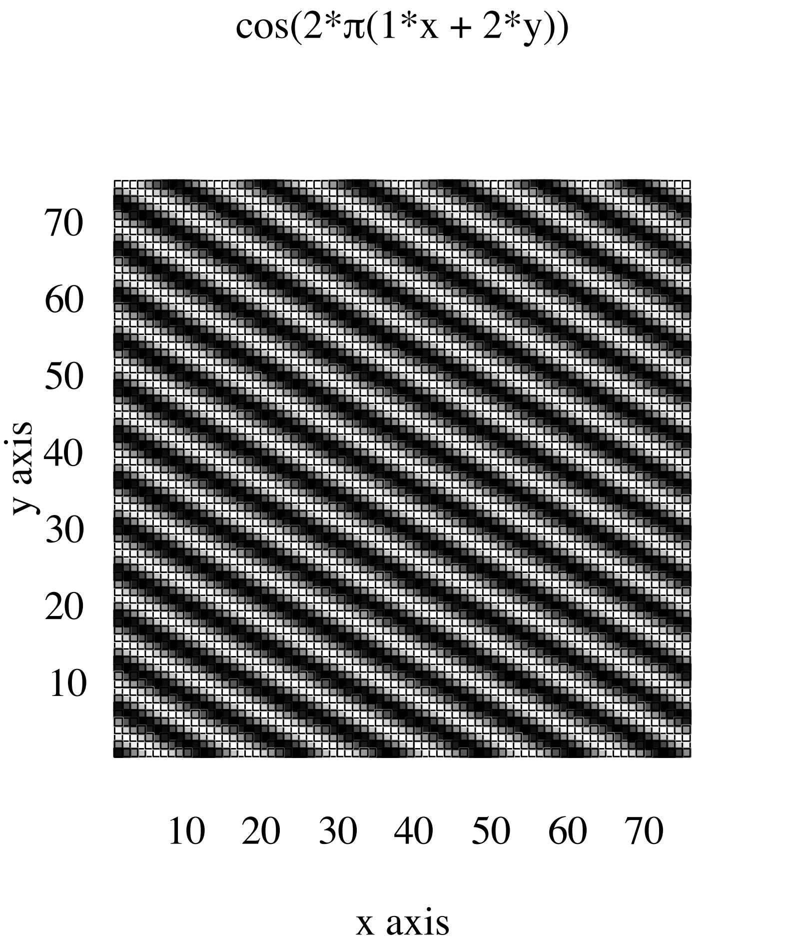
Further

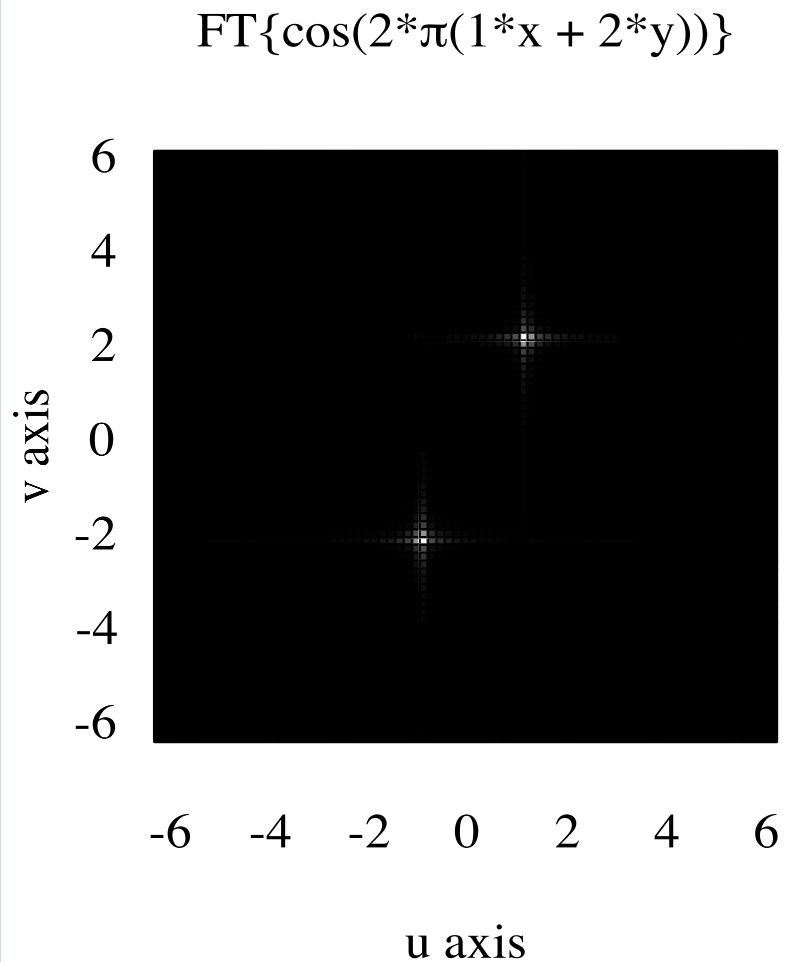


In the frequency domain these signals are represented by two impulses shifted by $-w_k$ and $+w_k$

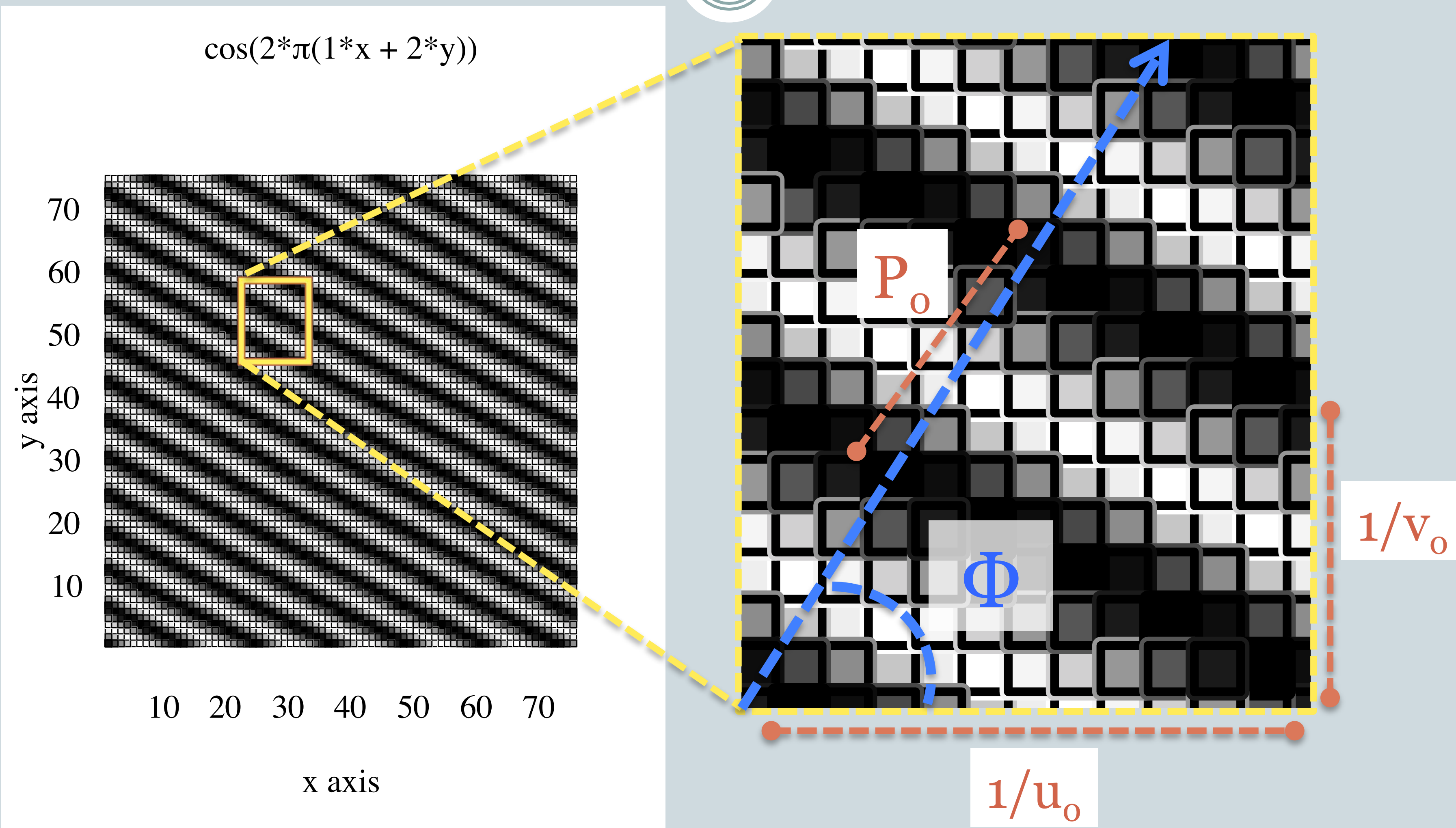
The w_k at this distance is so high, that the human observer cannot perceive the bars of the fence. The observer sees a wall, because those higher frequencies are cut by the low pass filter characteristic of the human visual system.

The reasoning behind CSFT

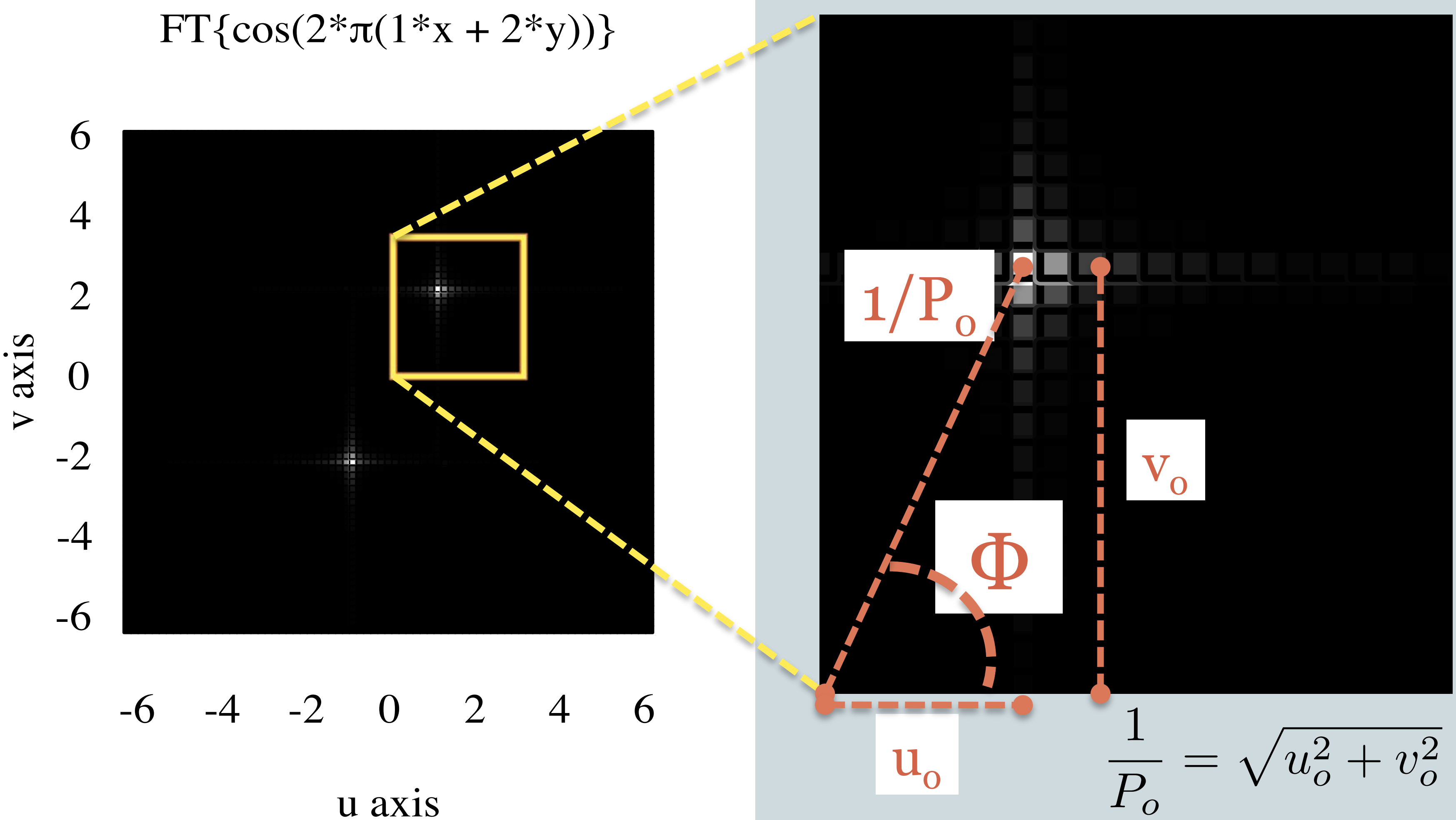




The reasoning behind CSFT (Cont.)



The reasoning behind CSFT (Cont.)



$$\frac{1}{P_o} = \sqrt{u_o^2 + v_o^2}$$

The reasoning behind CSFT (Cont.)

- Nice examples of CSFT on images
  - http://www.imagemagick.org/Usage/fourier/

The basics

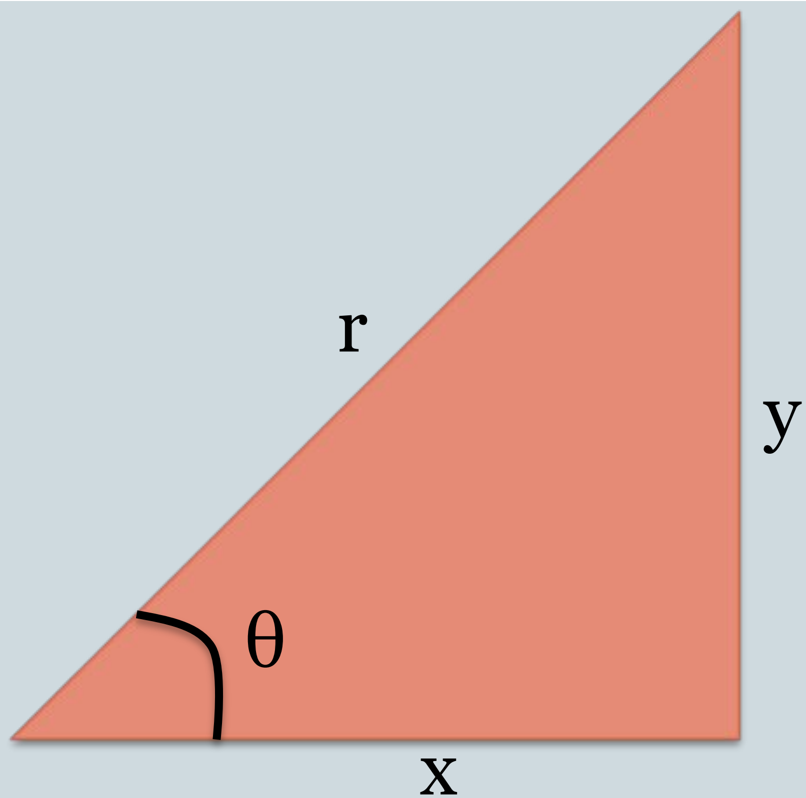
Let

$$(x, y) =: (r\cos(\theta), r\sin(\theta))$$

$$f(x, y) = \tilde{f}(r, \theta)$$

$$(u, v) =: (\rho\cos(\phi), \rho\sin(\phi))$$

$$F(u, v) = \tilde{F}(\rho, \phi)$$



Trigonometry

$$r^2 = x^2 + y^2,\ \cos(\theta) = \frac{x}{r},\ \sin(\theta) = \frac{y}{r}$$

From rectangular to polar coordinates

Recall:

$$F(u,v) = \iint_{\Re^2} f(x,y) e^{-j2\pi(ux+vy)} dx dy$$

Change of variables:

$$dx \cdot dy = r \cdot dr \cdot d\theta$$

$$\begin{aligned} ux + vy &= \rho\cos(\phi) r\cos(\theta) + \rho\sin(\phi) r\sin(\theta) \\ &= \rho \cdot r\left(\cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)\right) \\ &= \rho \cdot r \cdot \cos(\phi - \theta) \end{aligned}$$

Then:

$$\tilde{F}(\rho,\phi) = \int_0^{2\pi} \int_0^{\infty} \tilde{f}(r,\theta) e^{-j2\pi\rho r\cos(\phi-\theta)} r \cdot dr d\theta$$

$$\tilde{f}(r,\theta) = \int_0^{2\pi} \int_0^{\infty} \tilde{F}(\rho,\phi) e^{j2\pi\rho r\cos(\phi-\theta)} \rho \cdot d\rho d\phi$$

Properties

Rotation:

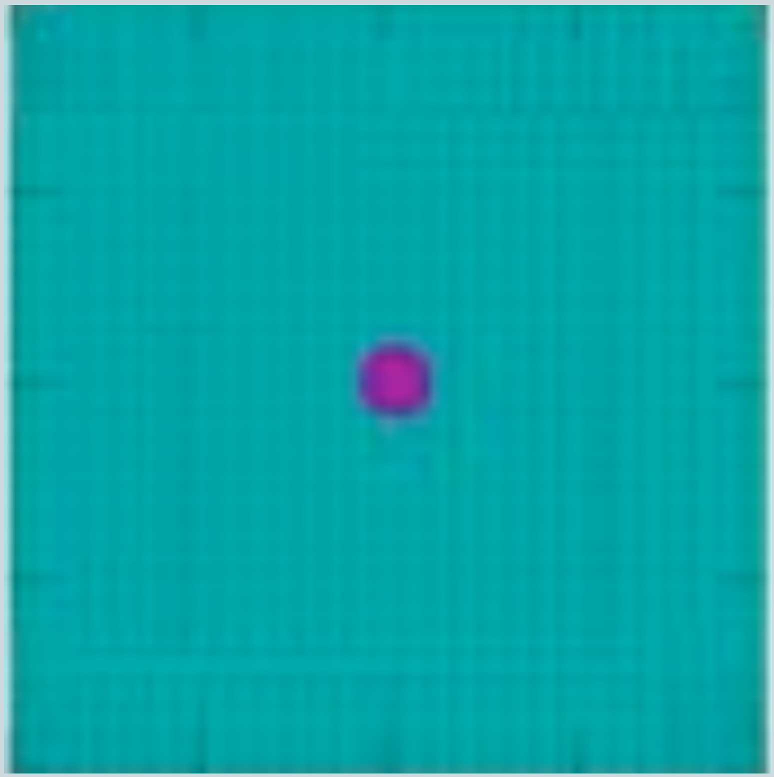
$$\tilde{f}(r, \theta + \theta_o) \xrightarrow{\mathcal{F}} \tilde{F}(\rho, \phi + \theta_o)$$

*proof by variable substitution z = $\theta - \theta_o$

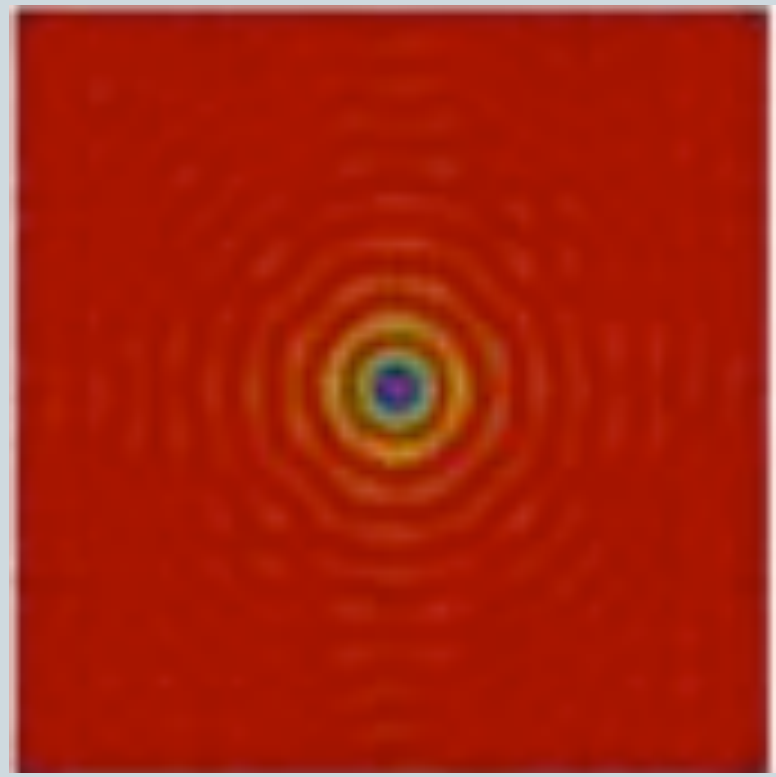
Circular symmetry:

$$\tilde{f}(r, \theta) = \tilde{f}_o(r) \Leftrightarrow \tilde{F}(\rho, \phi) = \tilde{F}_o(\rho)$$

Example of circular symmetry



circ(x,y)



jinc(u,v)