Unit Impulse & Unit Steps

Tuesday, August 28, 2007
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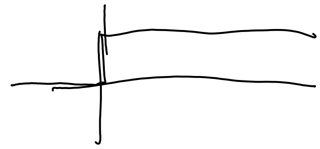
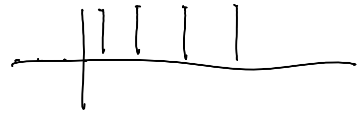
CT unit step

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$



DT unit step

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$



CT unit impulse (dirac delta)

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases} \qquad \int \delta(t)\, dt = 1$$

DT

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

Relationships between unit step + impulse

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

$$\delta[n] = u[n] - u[n-1]$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau = \int_{0}^{\infty} \delta(t-\tau)\, d\tau$$

$$\delta(t) = \frac{d}{dt} u(t)$$

Unit impulse is useful for sampling

∞

$$x(0) = \int_{-\infty} x(t)\,\delta(t)\,dt$$

because $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} x(0)\,\delta(t)\,dt = x(0)\int_{-\infty}^{\infty} \delta(t)\,dt = x(0)\cdot 1$

More Generally : $x(t_0) = \int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt$

because $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = \int_{-\infty}^{\infty} x(t_0)\,\delta(t-t_0) = x(t_0)\int_{-\infty}^{\infty} \delta(t-t_0)\,dt = x(t_0)$

DT Analogue

$x[0] = x[n]\,\delta[n]$

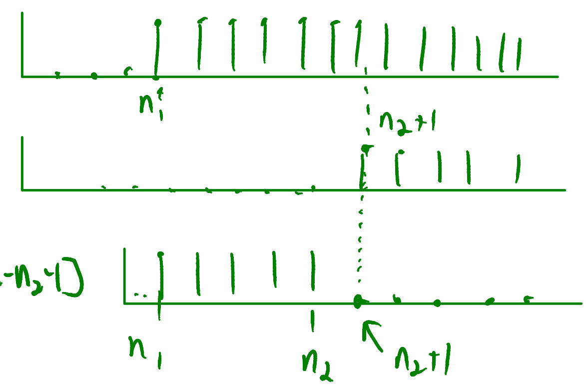
$x[n_0] = x[n]\,\delta[n-n_0]$

let $n_2 > n_1$

$u[n-n_1]$

$u[n-n_2-1]$

$u[n-n_1] - u[n-n_2-1]$



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