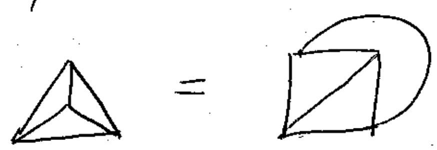
27 March 2012

Isomorphism



a b

Problem: 2 graphs, G & H, defined by a pic, matrix, adjmatrix, ...

Q: Determine whether they are the same.

In some sense, easy answer to the solution could be tryout out all possible labels to the second graph and see if it corresponds to the first, ~~of~~ labeled in some arbitrary way.

<u>Fact</u> There are $|H|!$ labelings

Γ Recall $\det(A) = \underbrace{\sum_{\sigma \text{ perm}}}_{n! \text{ terms}} (-1)^{\operatorname{sgn}(\sigma)} \cdot a_{1\sigma(1)} \cdot a_{2\sigma(2)} \cdots a_{n\sigma(n)}$

However, finding the rref(A) can be computed in $\sim \frac{2}{3}n^3$ steps ($< n!$)

<u>Moral</u>: Some problems look "hard" but can be "easy"

Γ Given $Ax = b$

it is somewhat easier to test a suggested solution x_0 for fitting the equation than to solve from scratch.

solving: $\sim \frac{n^3}{3}$ steps

testing: $\sim 2n^2$ steps

] (Whereabout these conc come from is not important)

<u>Def</u>: The class of problems that can be stated with n bits of information and can be solved (w/ turing machine) in polynomial time is called "P"

→ we have little idea of what kinds of things lives in P and what does not.

Def: The class of problems that can be stated in n bits of information and of solutions that can be checked in polynomial time is "NP"

BIG QUESTION! Is $P = NP$?

Digression: ~100yr ago, David Hilbert was the King of Mathematics who knew everything about mathematics. (such claim is impossible to make in ever large mathematical landscape of today). He proposed 20 or so questions he had not known the answer to; while ~1/0 were solved within 10 yrs of his stating of the problems, the rest remains as puzzling as it was 100 yrs ago. $P=NP$ is one of such.

It is obviou that $NP \supseteq P$ (or is it?)

Fact We don't know $NP = P$. But there exists problems Q in NP s.t. if we could show $Q \in P$, then it must follow that $P = NP$.

Q = the utterly difficult question ... the "NP-Complete".

Fact Testing graph isomorphy is NP-complete.

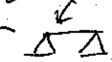
Motto: should look for criteria that says yes or no.

In order to certify that $G \cong H$ (isomorphic) you need to find a labeling of both graphs G + H such that the adjacency matrices become equal.

suppose: $G \cong H$. What kinds of facts follow?

- Same number of vertices
- — " — edges
- same list of degree of vertices
- connected components
- same # of cut edges

cut-edge

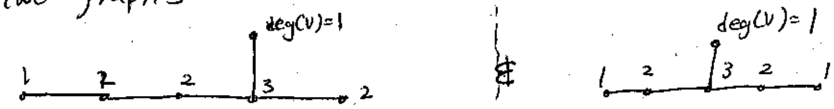


- — " — simple circuits of length 3, 4

- Same subgraphs of vertices of degree $\geq k$ & $\leq k$ in rang $[a, b]$

e.g.

two graphs



Violation of any of the above rules implies that G & H are <u>not</u> isomorphic. On the other hand, positive result says nothing ... only finding of particular labeling of vertices so that G & H produce adj. matrices is the only time when we can say that it is, indeed, isomorphic.

<u>Euler & Hamilton Graphs</u>

<u>Euler Graphs</u>

<u>Def</u>: An Euler Path in G is a path that uses every edge exactly once.

An Euler Circuit is an Euler path whose terminus is also its starting point.

What conditions are necessary for the existence of Euler circuit?

(1) Each vertex, besides initial/terminus vertices must have an even degree.

(2) If term/initial vertex is indistinct, they should have <u>odd degree</u>

(3) If they do not agree, the degree of this initial/terminus should be even

(4) G should be connected.

<u>Thm</u>: G has prop (1), (2), (3), (4) $\Longleftrightarrow$ G has an Euler Path.

<u>Fleury's Algorithm</u>

Input: G satisfies the above four properties.

Output: An Euler Path in G

outline

0. step: Position yourself at the initial vertex (if all vertexes are even, pick any even degree vertex) at odd if it exists

1. : Choose an unused edge and "use" it. Deleted the used edge. that is
→ not a cut-edge