ECE 544 Fall 2013
Problem Set 4
Due September 23, 2013

1. Read Chapter 5 of M. B. Pursley, *Introduction to Digital Communications* (MBP).
2. MBP Problems 4.9, 5.3, 5.6
3. Ziemer & Tranter, *Principles of Communications* (ZT), Problem A.5.

A.5. Assuming a bandwidth of 2 MHz, find the rms noise voltage across the output terminals of the circuit shown in Figure A.9 if it is at a temperature of 400 K.

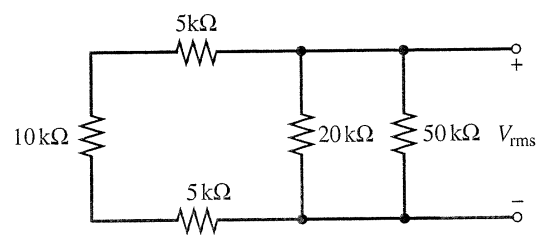

Figure A.9

4. ZT Problem A.7.

A.7. A source with equivalent noise temperature $T_s = 1000$ K is followed by a cascade of three amplifiers having the specifications shown in Table A.1. Assume a bandwidth of 50 kHz.

Table A.1

Amplifier no.	F	T_e	Gain
1		300 K	10 dB
2	6 dB		30 dB
3	11 dB		30 dB

a. Find the noise figure of the cascade.

b. Suppose amplifiers 1 and 2 are interchanged. Find the noise figure of the cascade.

c. Find the noise temperature of the systems of parts (a) and (b).

d. Assuming the configuration of part (a), find the required input signal power to give an output SNR of 40 dB. Perform the same calculation for the system of part (b).

5. ZT Problem A.12 (skip part (d)).

A.12. Given a relay–user link as described in Section A.3 with the following parameters:

Average transmit power of relay satellite: 35 dBW
Transmit frequency: 7.7 GHz
Effective antenna aperture of relay satellite: 1 m^2
Noise temperature of user receiver (including antenna): 1000 K
Antenna gain of user: 6 dB
Total system losses: 5 dB
System bandwidth: 1 MHz
Relay–user separation: 41,000 km

a. Find the received signal power level at the user in dBW.

b. Find the receiver noise level in dBW.

c. Compute the SNR at the receiver in decibels.

d. Find the average probability of error for the following digital signaling methods: (1) BPSK, (2) binary DPSK, (3) binary noncoherent FSK, (4) QPSK.[4]

Relay-User Link Example of Section A.3

A.3 FREE-SPACE PROPAGATION EXAMPLE

As a final example of noise calculation, we consider a free-space electromagnetic-wave propagation channel. For the sake of illustration, suppose the communication link of interest is between a synchronous-orbit relay satellite and a low-orbit satellite or aircraft, as shown in Figure A.7.

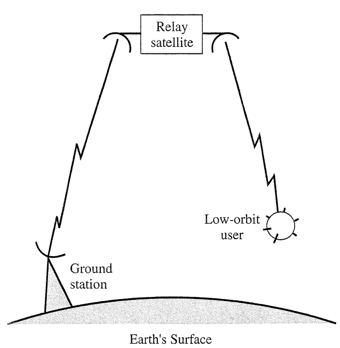

Figure A.7
A satellite-relay communication link.

This might represent part of a relay link between a ground station and a small scientific satellite or an aircraft. Since the ground station is high power, we assume the ground-station-to-relay-satellite link is noiseless and focus our attention on the link between the two satellites.

Assume a relay satellite transmitted signal power of P_T W. If radiated isotropically, the power density at a distance d from the satellite is given by

$$p_t = \frac{P_T}{4\pi d^2} \text{ W/m}^2 \tag{A.66}$$

If the satellite antenna has directivity, with the radiated power being directed toward the low-orbit vehicle, the antenna can be described by an antenna power gain G_T over the isotropic radiation level. For aperture type antennas with aperture area A_T large compared with the square of the transmitted wavelength λ^2, it can be shown that the maximum gain is given by $G_T = 4\pi A_T/\lambda^2$. The power P_R intercepted by the receiving antenna is given by the product of the receiving aperture area A_R and the power density at the aperture. This gives

$$P_R = \frac{P_T G_T}{4\pi d^2} A_R \tag{A.67}$$

However, we may relate the receiving aperture antenna to its maximum gain by the expression $G_R = 4\pi A_R/\lambda^2$, giving

$$P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi d)^2} \tag{A.68}$$

Equation (A.68) includes only the loss in power from isotropic spreading of the transmitted wave. If other losses such as atmospheric absorption are important, they may be included as a loss factor L_0 in (A.68) to yield

$$P_R = \left(\frac{\lambda}{4\pi d}\right)^2 \frac{P_T G_T G_R}{L_0} \tag{A.69}$$

The factor $(4\pi d/\lambda)^2$ is sometimes referred to as the *free-space loss*.[3]

In the calculation of receiver power, it is convenient to work in terms of decibels. Taking $10 \log_{10} P_R$, we obtain

$$\begin{aligned} 10 \log_{10} P_R &= 20 \log_{10}\left(\frac{\lambda}{4\pi d}\right) + 10 \log_{10} P_T \\ &\quad + 10 \log_{10} G_T + 10 \log_{10} G_R - 10 \log_{10} L_0 \end{aligned} \tag{A.70}$$

Now $10 \log_{10} P_R$ can be interpreted as the received power in decibels referenced to 1 W; it is commonly referred to as power in decibel watt. Similarly, $10 \log_{10} P_T$ is commonly referred to as the transmitted signal power in decibel watt. The terms $10 \log_{10} G_T$ and $10 \log_{10} G_R$ are the transmitter and receiver antenna gains (above isotropic) in decibels, while the term $10 \log_{10} L_0$ is the loss factor in decibels. When $10 \log_{10} P_T$ and $10 \log_{10} G_T$ are taken together, this sum is referred to as the *effective radiated power* in decibel watts (ERP, or sometimes EIRP, for effective radiated power referenced to isotropic). The negative of the first term is the free-space loss in decibels. For $d = 10^6$ mi $(1.6 \times 10^9 \text{ m})$ and a frequency of 500 MHz $(\lambda = 0.6 \text{ m})$,

$$20 \log_{10}\left(\frac{\lambda}{4\pi d}\right) = 20 \log_{10}\left(\frac{0.6}{4\pi \times 1.6 \times 10^9}\right) = -210 \text{ dB} \tag{A.71}$$

[3]We take the convention here that a *loss* is a factor in the denominator of P_R; a *loss* in decibels is a positive quantity (a negative gain).

If λ or d change by a factor of 10, this value changes by 20 dB. We now make use of (A.70) and the results obtained for noise figure and temperature to compute the SNR for a typical satellite link.

EXAMPLE A.8

We are given the following parameters for a relay-satellite-to-user link:

Relay satellite effective radiated power ($G_T = 30$ dB; $P_T = 100$ W): 50 dBW

Transmit frequency: 2 GHz ($\lambda = 0.15$ m)

Receiver noise temperature of user (includes noise figure of receiver and background temperature of antenna): 700 K

User satellite antenna gain: 0 dB

Total system losses: 3 dB

Relay–user separation: 41,000 km

Find the signal-to-noise power ratio in a 50 kHz bandwidth at the user satellite receiver IF amplifier output.

Solution

The received signal power is computed using (A.69) as follows ($+$ and $-$ signs in parentheses indicate whether the quantity is added or subtracted):

Free-space loss: $-20\log_{10}(0.15/4\pi \times 41 \times 10^6)$: 190.7 dB ($-$)

Effective radiated power: 50 dBW ($+$)

Receiver antenna gain: 0 dB ($+$)

System losses: 3 dB ($-$)

Received Signal Power: -143.7 dBW

The noise power level, calculated from (A.43), is

$$P_{\text{int}} = G_a k T_e B \tag{A.72}$$

where P_{int} is the receiver output noise power due to internal sources. Since we are calculating the SNR, the available gain of the receiver does not enter the calculation because both signal and noise are multiplied by the same gain. Hence, we may set G_a to unity, and the noise level is

$$\begin{aligned} P_{\text{int, dBW}} &= 10\log_{10}\left[kT_0\left(\frac{T_e}{T_0}\right)B\right] \\ &= 10\log_{10}(kT_0) + 10\log_{10}\left(\frac{T_e}{T_0}\right) + 10\log_{10} B \qquad \text{(A.73)} \\ &= -204 + 10\log_{10}\left(\frac{700}{290}\right) + 10\log_{10} 50{,}000 \\ &= -153.2\ \text{dBW} \end{aligned}$$

Hence, the SNR at the receiver output is

$$SNR_0 = -143.7 + 153.2 = 9.5\ \text{dB} \tag{A.74}$$

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EXAMPLE A.9

To interpret the result obtained in the previous example in terms of the performance of a digital communication system, we must convert the SNR obtained to energy-per-bit-to-noise-spectral density ratio E_b/N_0 (see Chapter 8). By definition of SNR_0, we have

$$\mathrm{SNR}_0 = \frac{P_R}{kT_eB} \tag{A.75}$$

Multiplying numerator and denominator by the duration of a data bit T_b, we obtain

$$\mathrm{SNR}_0 = \frac{P_RT_b}{kT_eBT_b} = \frac{E_b}{N_0BT_b} \tag{A.76}$$

where $P_RT_b = E_b$ and $kT_e = N_0$ are the signal energy per bit and the noise power spectral density, respectively. Thus, to obtain E_b/N_0 from SNR_0, we calculate

$$\left.\frac{E_b}{N_0}\right|_{\mathrm{dB}} = (\mathrm{SNR}_0)_{\mathrm{dB}} + 10\log_{10}(BT_b) \tag{A.77}$$

For example, from Chapter 8 we recall that the null-to-null bandwidth of a phase-shift keyed carrier is $2/T_b$ Hz. Therefore, BT_b for BPSK is 2 (3 dB) and

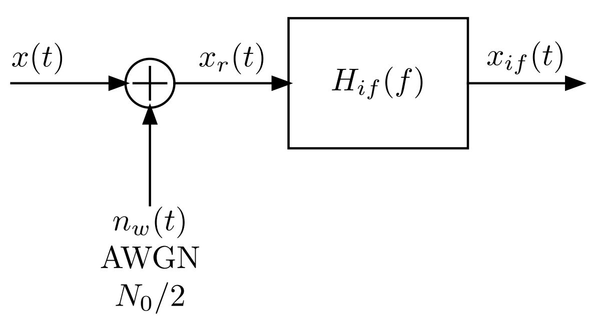
$$\left.\frac{E_b}{N_0}\right|_{\mathrm{dB}} = 9.5 + 3 = 12.5 \text{ dB} \tag{A.78}$$

The probability of error for a binary BPSK digital communication system was derived in Chapter 8 as

$$\begin{aligned} P_E &= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \cong Q\left(\sqrt{2\times 10^{1.25}}\right) \\ &\cong 1.23\times 10^{-9} \quad \text{for} \quad \frac{E_b}{N_0} = 12.5 \text{ dB} \end{aligned} \tag{A.79}$$

which is a fairly small probability of error (anything less than 10^{-6} would probably be considered adequate). It appears that the system may have been overdesigned. However, no margin has been included as a safety factor. Components degrade or the system may be operated in an environment for which it was not intended. With only 3 dB allowed for margin, the performance in terms of error probability becomes 1.21×10^{-5}.

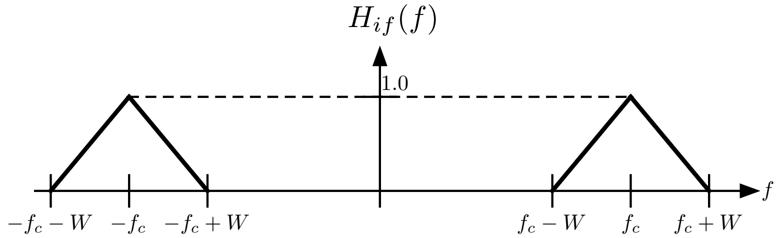
6. The block diagram below shows the front end of an AM DSB communication system.



The input signal is an AM DSB waveform where the message is sinusoidal, i.e.,

$$x(t) = A_cA_m\cos(2\pi f_mt)\cos(2\pi f_ct)$$

($f_m \ll f_c$). We model $x(t)$ as a deterministic power signal for simplicity. The triangularly shaped IF filter $H_{if}(f)$ has bandwidth $2W$ where $0 < W < f_c$. The goal of the problem is to choose W to maximize SNR_T, the ratio of the power of the signal part in $x_{if}(t)$ to the power of the noise part in $x_{if}(t)$.



(a) Signal:

- Find the signal component in the output of the IF filter, i.e., $x * h_{if}(t)$, as a function of W, $0 < W < f_c$. It is helpful to consider two cases: 1) when $0 < W < f_m$ and 2) when $f_m \leq W < f_c$.
- Then find the power of the above signal component in terms of the fixed parameters A_c, A_m, f_m and the variable W.
- Make a rough sketch of this power as a function of W, $0 < W < f_c$, and comment.

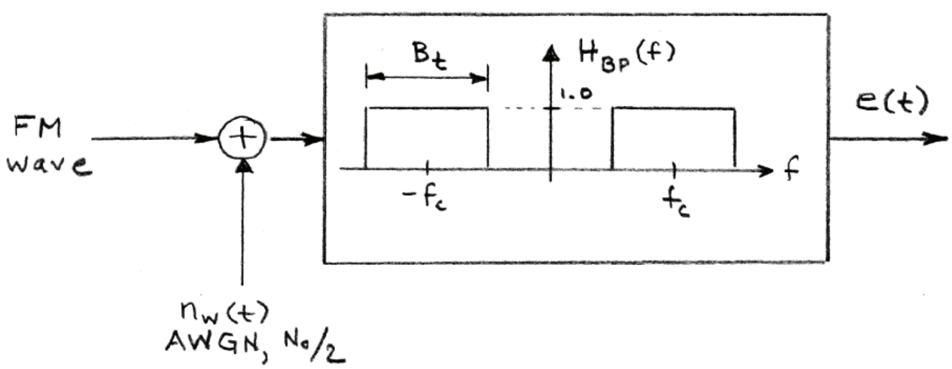
(b) Find the power of the noise component in the output of the IF filter, i.e., $n_w * h_{if}(t)$, as a function of W, $0 < W < f_c$, roughly plot it, and comment.

(c) Now find SNR_T as defined above and comment on its general behavior as a function of W, $0 < W < f_c$.

(d) Find the value W_* maximizing SNR_T as a function of W, $0 < W < f_c$.

(e) Let SNR_{T*} be the maximum SNR obtained in (d). What is the degradation in dB of this triangular IF filtering scheme compared to the best one could do with the "classical" rectangular IF filters we examined in class?

7. The model for analysis of FM discriminator detection in AWGN assumes a receiver front end consisting of a bandpass filter coming before the discriminator. This front end is shown in the figure below. The BW of the bandpass filter is B_t and is set equal to a value needed to pass the FM wave with minimal distortion.



(a) The signal $e(t)$ at the output of the bandpass filter is the sum of the FM wave of power $P_T = A_c^2$ and the filtered noise $n(t) = [h_{BP} * n_w](t)$. Sketch the power spectral density $S_{n,n}(f)$ of the filtered noise. Compute the power in the noise $n(t)$ and write down the SNR at the output of the bandpass filter.

(b) Let the in-phase/quadrature expansion of $n(t)$ be

$$n(t) = \sqrt{2}n_I(t)\cos(2\pi f_c t) - \sqrt{2}n_Q(t)\sin(2\pi f_c t)$$

where $n_L(t) = n_I(t) + jn_Q(t)$ is the complex envelope of $n(t)$. Find and sketch the power spectral densities of $n_L(t)$, $n_I(t)$, and $n_Q(t)$. Compute the power in each of these random processes. For the scenario given here, what is the cross-correlation function between the in-phase and quadrature components of the noise $n_I(t)$ and $n_Q(t)$?

(c) In the class notes we derived the following expression for the SNR at the output of a discriminator demodulator

$$\mathrm{SNR}_D = K \left(\frac{B_t}{W}\right)^2 \left(\frac{P_T}{N_0 W}\right).$$

In the above equation W is the baseband bandwidth of the message used to modulate the FM wave and B_t/W is the bandwidth expansion factor.

In commercial FM broadcast the maximum frequency deviation of an FM wave is limited to 75 kHz by FCC regulation. The message bandwidth is $W = 15$ kHz. Using Carson's rule find the transmission bandwidth B_t. For these parameters find the factor by which demodulated SNR is improved relative to baseband transmission (express both as a ratio and in dB). You may assume $K = 1$ for simplicity.

(d) As mentioned in class the expression for SNR_D would seem to indicate that the improvement over baseband transmission can be made arbitrarily large by increasing the transmission bandwidth. However, the large SNR assumption used to derive the expression will eventually break down. The class analysis was based on the assumption that $A_c \gg |n_L(t)|$. Recall that

$$R = |n_L(t)| = \sqrt{n_I^2(t) + n_Q^2(t)}$$

is a Rayleigh random variable with pdf

$$f_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} \quad \text{for } r \geq 0$$

where $\sigma^2 = \mathrm{Power}\{n_I(t)\}$. Suppose that we define the high SNR region to be the case where

$$\Pr\{|n_L(t)| \leq 0.1 A_c\} \geq 0.9.$$

Use this definition of high SNR to find a lower bound on $P_T/(N_0 W)$ above which the class expression for SNR_D holds. The lower bound will be of the form

$$\text{some simple function of } \left(\frac{B_t}{W}\right) \leq \frac{P_T}{N_0 W}.$$

(e) Evaluate the lower bound for the broadcast FM parameters of Part (c).