(22 pts) **1.** Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

$y(1) = x(2)$
$y(-1) = x(-2)$

$y(2) = x(-4)$
$y(-2) = x(-4)$
$\cancel{y(-1) = x(-1)}$
$y(-\frac{1}{2}) = x(-\frac{1}{4})$
Future

| | Yes | No |
|---|---|---|
| If $y(t) = x(2t)$, is the system causal? | | ✓ |
| If $y(t) = (t+2)x(t)$, is the system causal? | ✓ | |
| If $y(t) = x(-t^2)$, is the system causal? | | ✓ |
| If $y(t) = x(t) + t - 1$, is the system memoryless? | ✓ | |
| If $y(t) = x(t^2)$, is the system memoryless? $y(3) = x(9)$ | | ✓ |
| If $y(t) = x(t/3)$, is the system stable? | ✓ | |
| – If $y(t) = tx(t/3)$, is the system stable? | | ✓ |
| If $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, is the system stable? | | ✓ |
| – If $y(t) = \sin(x(t))$, is the system time invariant? | | ✓ X |
| – If $y(t) = u(t) * x(t)$, is the system LTI? | ✓ | |
| – If $y(t) = (tu(t)) * x(t)$, is the system linear? | ✓ | |

$\underline{\sin(x(t))}$

$x(t) \to \text{System} \to y(t) = \sin(x(t)) \to TD \to z(t) = y(t-t_0) \Rightarrow z = \sin(x(t-t_0))$

$x(t) \to TD \to g(t) = x(t-t_0) \to \text{system} \to z(t) = y(\sin(x(t)))$

$= x(\sin(x(t)) - t_0)$

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(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t+2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all $\sum$ signs disappear.)

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\, u(t-\tau+2)\, d\tau$$

$$= \int_{0}^{\infty} e^{-\tau} u(t-\tau+2)\, d\tau \qquad \text{since } u(\tau) = 0 \text{ for } \tau < 0$$

$t-\tau+2 \geq 0$ ; $\tau \leq t+2$ ; $t+2 \geq 0$ ; $t \geq -2 \Rightarrow u(t+2)$

$$= \left( \int_{0}^{t+2} e^{-\tau}\, d\tau \right) u(t+2)$$

$$= \left[ -e^{-\tau} \right]_{0}^{t+2} u(t+2)$$

$$= \left[ -e^{-(t+2)} + e^{0} \right] u(t+2)$$

$$y(t) = \left[ 1 - e^{-t-2} \right] u(t+2)$$

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(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$x(t) = \frac{3e^{jt}}{1+j} = \frac{3e^{jt}}{\sqrt{2}\,\angle 45^\circ}$$

$e^{j\frac{\pi}{4}}$

$$|x(t)| = \frac{3}{\sqrt{2}}$$

-3

$$\therefore E_\infty = \int_{-\infty}^{\infty} \left(\frac{\sqrt{3}}{2}\right)^2 dt = \int_{-\infty}^{\infty} \frac{3}{4}\, dt = \left[\frac{3}{4}t\right]_{-\infty}^{\infty} \Rightarrow \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{3}{4}\, dt = \lim_{T\to\infty} \frac{1}{2T} \left[\frac{3}{4}t\right]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{3}{4}T + \frac{3}{4}T\right] = \lim_{T\to\infty} \frac{1}{2T} \cdot \left(\frac{3}{2}T\right) = \frac{3}{4}$$

2

(15 pts) **4.** Compute the coefficients $a_k$ of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

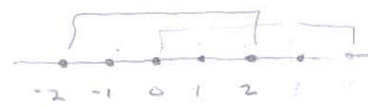
$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}.$$

(Simplify your answer as much as possible.)

$$\sin(\pi t) = \frac{e^{j\pi t} - e^{-j\pi t}}{2j}$$

$$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\left(\frac{2\pi}{T}\right)t} dt$$



$$\sin(\pi t) = \frac{1}{2j} e^{j\pi t} - \frac{1}{2j} e^{-j\pi t}$$

$$\Rightarrow \frac{1}{2j} e^{j\pi t} - \frac{1}{2j} e^{j\pi t} = 0$$

Also,
$x(0) = 0$
$x(1) = 0$
$x(2) = 0$
$x(3) = 0$

$$a_0 = \frac{1}{4} \int_0^2 \sin(\pi t) e^0 \, dt + \frac{1}{4} \int_2^4 0 \, dt = \frac{1}{4} \int_0^2 \sin(\pi t) \, dt$$

$$= \frac{1}{4} \left[ -\frac{1}{\pi} \cos(\pi t) \right]_0^2 = \frac{-1}{4\pi} \left[ \cos(2\pi) - \cos(0) \right] = \frac{-1}{4\pi} [1 - 1] = 0$$

$\therefore a_0 = 0.$



$\therefore a_0 = 0, \ a_1 = 0, \ a_2 = 0, \ a_3 = 0$

**5.** A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

$$
\begin{array}{rcl}
\text{input} & & \text{output} \\
x_0[n] = \delta[n] & \rightarrow & y_0[n] = \delta[n-1], \\
x_1[n] = \delta[n-1] & \rightarrow & y_1[n] = 4\delta[n-2], \\
x_2[n] = \delta[n-2] & \rightarrow & y_2[n] = 9\delta[n-3], \\
x_3[n] = \delta[n-3] & \rightarrow & y_3[n] = 16\delta[n-4], \\
 & \vdots & \\
x_k[n] = \delta[n-k] & \rightarrow & y_k[n] = (k+1)^2\delta[n-(k+1)] \text{ for any integer k.}
\end{array}
$$

(10 pts) **a)** Can this system be time-invariant? Explain.

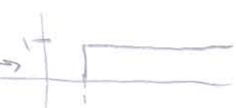
be precise.

5

No, because if you shift the input by a time $t_0$, the delta function gets shifted by $t_0$, BUT, a different coefficient is applied to the delta function, so they are not the same.

(10 pts) **b)** Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n-1]$?

$y[n] \rightarrow$



~~$x[n] = \frac{1}{(n+1)^2} u[n]$~~

0

Need the $\frac{1}{(n+1)^2}$ b/c of the increasing coeff. of output.