Due Wednesday March 11 at 5:00 PM

ECE 438 Assignment No. 6 Spring 2009

1. The signal $x(t) = 0.8\cos[2\pi(500)t] + 0.3\cos[2\pi(3500)t]$ is sampled at 10 kHz using an ideal A/D converter to produce the digital signal $x[n]$. You compute a 512-point DFT $X[k]$ of this signal. Find the approximate values of k and the amplitudes $|X[k]|$ corresponding to the spectral peaks in the analog signal.

2. Consider the length N signal x[n] $\neq 0$, only for n=0, 1, …, N-1. Let y[n] be a new signal defined by padding x[n] with zeros to make it length M=LN for some integer L:

$$y[n] = \begin{cases} x[n], & n = 0,\ldots,N-1 \\ 0, & n = N,\ldots,M-1 \end{cases}$$

Let $X(\omega)$ be the DTFT of x[n] and let Y[k], k=0,…, M-1 be the M point DFT of y[n].

Show that Y[k] yields samples from $X(\omega)$ at points $\omega_k = 2\pi k / M$, k=0,1,…, M-1, i.e.

$$Y[k] = X\left(\frac{2\pi k}{M}\right), k = 0,\ldots,M-1 .$$

3. Consider the signal

$$x[n] = \cos(\omega_1 n) + a\cos(\omega_2 n) + b\, d[n],$$

where a and b are constants and $d[n]$ is a sequence of independent Gaussian random variables with zero mean and unit variance.

a. Write a MATLAB program that will

i. plot x[n],

ii. compute t he N point DFT $X[k]$ (using FFT routines available within MATLAB),

iii. plot $|X[k]|$.

Turn in a printout of your M-file with your homework.

b. Run your program and generate output for the cases shown in the table below. Turn in the plots generated for each case.

c. Discuss the significance of each case.

Case	N	ω_1	a	ω_2	b
1	20	0.62831853	0.0	-	0.0
2	200	0.62831853	0.0	-	0.0
3	20	0.64402649	0.0	-	0.0
4	200	0.64402649	0.0	-	0.0
5	200	0.64402649	0.2	1.27234502	0.0
6	200	0.64402649	0.2	0.79168135	0.0
7	200	0.64402649	0.2	0.79168135	0.05
8	200	0.64402649	0.2	0.79168135	0.2

4. a. Derive a decimation-in-time FFT algorithm for a 12 point DFT, and draw a *complete* flow diagram for the algorithm.

   b. Calculate the approximate number of complex operations required to compute the 12 point FFT and compare with the number of complex operations required to compute the 12 point DFT directly.

5. You have software to compute the forward DFT, but no software to compute the inverse DFT. Devise an approach to using your forward FFT software to actually compute the inverse DFT by first preprocessing the data, taking a forward DFT, then performing some postprocessing on the output from the FFT algorithm.

6. You want to exactly compute an exact 5120-point DFT. You have a radix 2 FFT subroutine that computes the DFT for $N = 2^M$ points for any integer value of M.

   a. Show how to use this subroutine to efficiently compute a 5120-point DFT.

   b. Draw a block diagram for your algorithm, showing the radix 2 FFT subroutine as a black box with no detail regarding what is inside it.

   c. Calculate the approximate number of complex operations required to compute the 5120-point DFT using your efficient approach, and compare with the number of complex operations required to compute the 5120-point DFT directly.