System Examples : Proof

e.g. 1) $y(t) = x(at)$ $\qquad x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(at)$

$x(t-t_0)$

Linear? Yes

$$a_1x_1(t) + a_2x_2(t) \rightarrow \boxed{S} \rightarrow y(t) = a_1x_1(at) + a_2x_2(at)$$

$$= a_1y_1(t) + a_2y_2(t)$$

where:

$$x_i(t) \rightarrow \boxed{S} \rightarrow y_i(t) = x_i(at)$$

$i=1,2$

TI? No

$x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(at)$ $\quad x(at-t_0)$

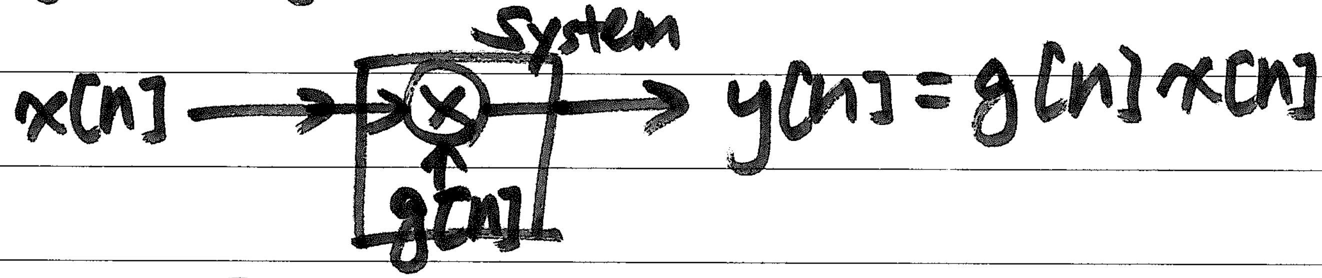
$x(t-t_0) \rightarrow \boxed{S} \rightarrow z(t) = x(at-t_0)$

Is $z(t) = y(t-t_0)$? No

$$\begin{array}{ccc} z(t) & & y(t-t_0) \\ \| & & \| \\ x(at-t_0) & & x(a(t-t_0)) = x(at-at_0) \end{array}$$

- System is stable.
- System is not causal if $a<0$
- DT system $y[n] = x[an]$ "same"

e.g.2) $y[n] = g[n]x[n]$



Linear? Yes

$$a_1 x_1[n] + a_2 x_2[n] \longrightarrow \boxed{S} \longrightarrow y[n]$$

$$= g[n]\{a_1 x_1[n] + a_2 x_2[n]\}$$

$$= a_1 \underbrace{g[n] x_1[n]}_{= y_1[n]} + a_2 \underbrace{g[n] x_2[n]}_{= y_2[n]}$$

$$= a_1 y_1[n] + a_2 y_2[n]$$

* due to distributive property of multiplication. It's linear.

"$y[n] = g[n]x[n]$"

TI? No

$$x[n-n_0] \longrightarrow \boxed{S} \longrightarrow z[n] = g[n]x[n-n_0]$$

Is $z[n] = y[n-n_0]$? No

$$\underset{g[n]x[n-n_0]}{\overset{}{z[n]}} \quad g[n]x[n-n_0] \neq g[n-n_0]x[n-n_0]$$

- system is stable if $|g[n]| < \infty$, $\forall n$
- system is causal ~~& mem~~
- system is memoryless

Comments

(w.r.t)

- Similar observations hold with respect to the CT system $y(t) = g(t)\,x(t)$
- Even though this system is not both Linear and Time-invariant (LTI), it is still very useful system in practice.
- The only ramification of not being LTI is that the output is not related to the input through convolution with the impulse response of the system.

LTI: $x(t) * h(t) = y(t)$: time-domain

$X(f)\,H(f) = Y(f)$: freq-domain

e.g. 3) Square-Law System

$$x(t) \rightarrow \boxed{S} \rightarrow y(t) = x^2(t)$$

Linear? No

$$a_1x_1(t) + a_2x_2(t) \rightarrow \boxed{S} \rightarrow y(t)$$

$$= \{a_1x_1(t) + a_2x_2(t)\}^2$$

$$= a_1^2x_1^2(t) + 2a_1a_2x_1(t)x_2(t) + a_2^2x_2^2(t)$$

$$\neq a_1x_1^2(t) + a_2x_2^2(t)$$

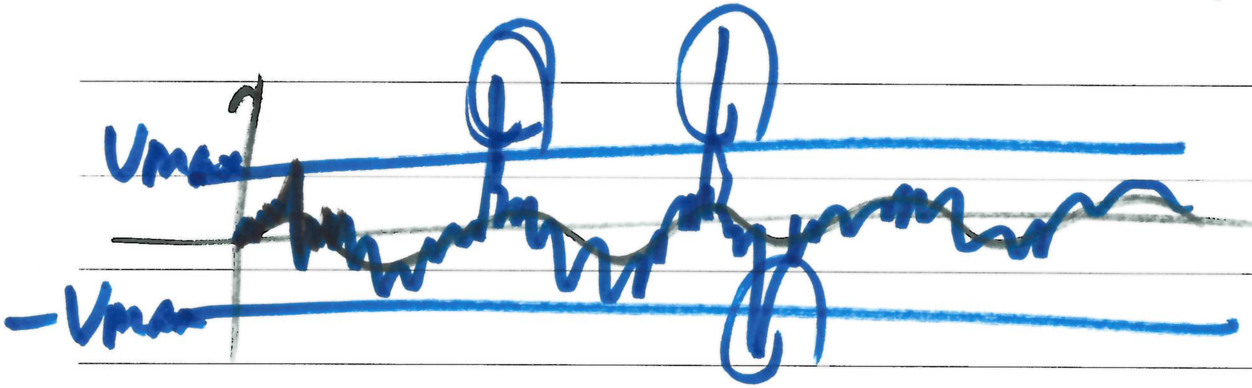
TI? Yes

$$x(t-t_0) \rightarrow \boxed{S} \rightarrow x^2(t-t_0)$$

- system is stable and causal
- similar observation holds w.r.t DT system $y[n] = x^2[n]$

e.g. 4) Hard-Limiter

$$y(t) = \begin{cases} V_{max}, & \text{if } x(t) > V_{max} \\ x(t), & \text{if } -V_{max} < x(t) < V_{max} \\ -V_{max}, & \text{if } x(t) < -V_{max} \end{cases}$$



- used to remove noisy spikes in radios (FM)
- used to fit signal into range of quantizer in process of A/D conversion after amplitude scaling.

e)

- System is TI but not Linear

> You can show nonlinearity with a single counter-example.

$x_1(t) = A_1 u(t)$ and $x_2(t) = A_2 u(t)$

, where $A_1 < V_{max}$, $A_2 < V_{max}$, but $A_1 + A_2 > V_{max}$

(2) (3) (2) (3) (4) (3)

> $x_1(t) \rightarrow \boxed{S} \rightarrow y_1(t) = A_1 u(t)$

$x_2(t) \rightarrow \boxed{S} \rightarrow y_2(t) = A_2 u(t)$

$x_1(t) + x_2(t) \rightarrow \boxed{S} \rightarrow y(t) = V_{max}\, u(t)$

$\neq y_1(t) + y_2(t)$

$= (A_1 + A_2)\, u(t)$

e.g. 5) Integrator

$$y(t) = \int_{t-T_b}^{t+T_a} x(\tau)\,d\tau$$: area under $x(t)$ $(t-T_b) \sim (t+T_a)$

, $T_a, T_b > 0$

LTI?