EE 301 **Exam 2** **3/10/05**

Name: ________________________

General Instructions:

- Write your name on every page of the exam.
- Do not write on the backs of the pages. If you need more paper, it will be provided to you upon request.
- The exam is closed book and closed notes. Calculators are **not** allowed or needed.
- A formula sheet will be handed out.
- Your work must be explained to receive full credit.
- Point values for each problem are as indicated. The exam totals 100 points.
- All plots must be carefully drawn with axes labeled.
- If you finish the exam during the first 50 minutes, you may turn it in and leave. During the last 10 minutes you must remain seated until we pick up exams from everyone.

This exam is for Krogmeier's section of 301.

Do not open the exam until you are told to begin.

Name: ______________________

Problem 1. [30 pts. total] Let $x(t)$ be a possibly complex-valued time signal with Fourier transform $X(j\omega)$. Express the Fourier transforms of the following time signals in terms of $X(j\omega)$:

(a) [7 pts.] $x^*(t)$.

(b) [7 pts.] $\text{Re}\{x(t)\}$.

(c) [8 pts.] $x(t-2) + x^*(-t-2)$.

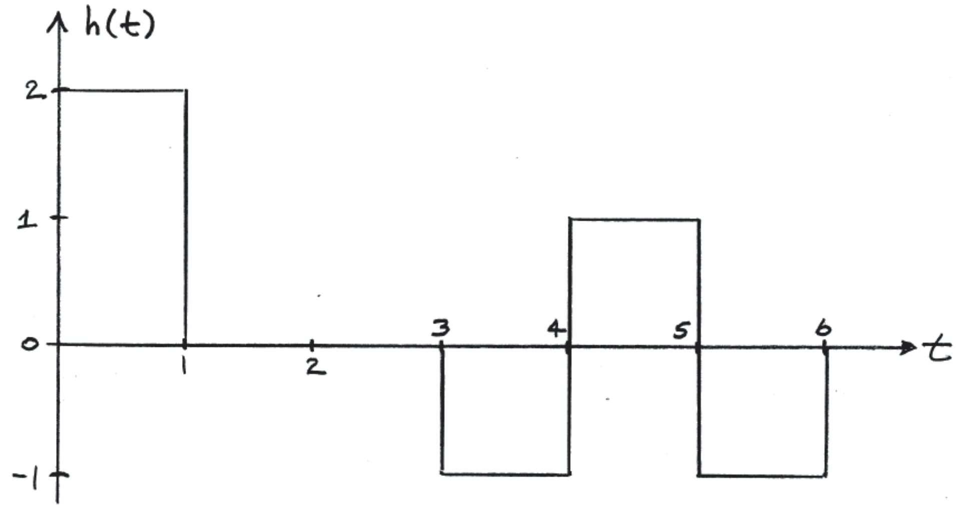
(d) [8 pts.] $e^{j4\pi t}x(t/3)$.

Name: ______________________

Problem 2. [20 pts. total] The impulse response $h(t)$ of an LTI filter is as given in the sketch below (note that $h(t) = 0$ for $t < 0$ and $t > 6$). Find the *equivalent noise bandwidth* of the filter, which is defined as

$$\text{BW} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|H(j\omega)|^2}{|H(j0)|^2} \mathrm{d}\omega$$

where $h(t) \leftrightarrow H(j\omega)$.

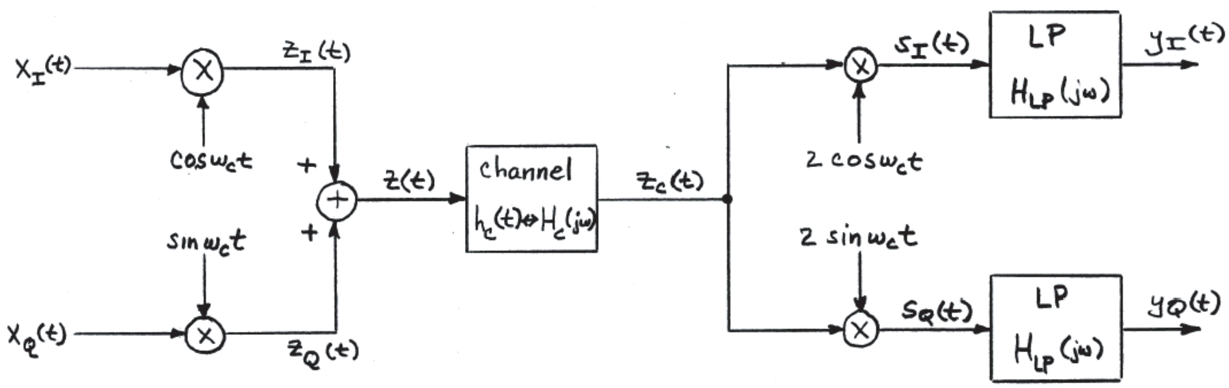


Name: ______________________

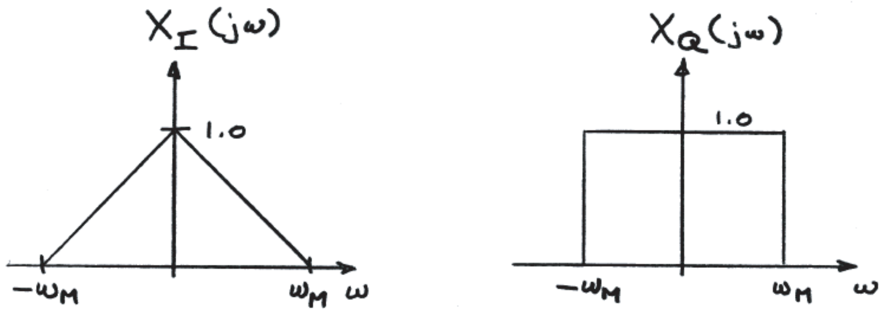
Problem 3. [50 pts. total] The block diagram below depicts the cascade of a quadrature modulator, an LTI communications channel $h_c(t) \leftrightarrow H_c(j\omega)$, and a quadrature demodulator. The low-pass filter blocks (labeled "LP") are identical, ideal low-pass filters with transfer function:

$$H_{LP}(j\omega) = \begin{cases} 1 & |\omega| \leq \omega_M \\ 0 & \text{otherwise} \end{cases} .$$

This is a method for sending two independent signals simultaneously over the same frequency channel. The point of this problem is to work through the analysis of this system in the frequency domain.



Assume that the two independent message waveforms $x_I(t) \leftrightarrow X_I(j\omega)$ and $x_Q(t) \leftrightarrow X_Q(j\omega)$ have the spectral shapes shown and that $\omega_M \ll \omega_c$. We assume, for simplicity in making later spectral plots, that both message spectra are *real-valued.*



For parts (a) – (c), the channel filter is assumed to have no effect, i.e., $h_c(t) = \delta(t)$**.**

(a) [15 pts.] Find expressions for $Z_I(j\omega)$, $Z_Q(j\omega)$, and $Z(j\omega)$ in terms of the two message spectra. Plot the real and imaginary parts of $Z(j\omega)$ on the axes provided.

(b) [15 pts.] Find expressions for $S_I(j\omega)$ and $S_Q(j\omega)$ and plot the real and imaginary parts of both on the axes provided.

(c) [10 pts.] From the plots, find $y_I(t) \leftrightarrow Y_I(j\omega)$ and $y_Q(t) \leftrightarrow Y_Q(j\omega)$.

Continued on next page.

Name: ________________________

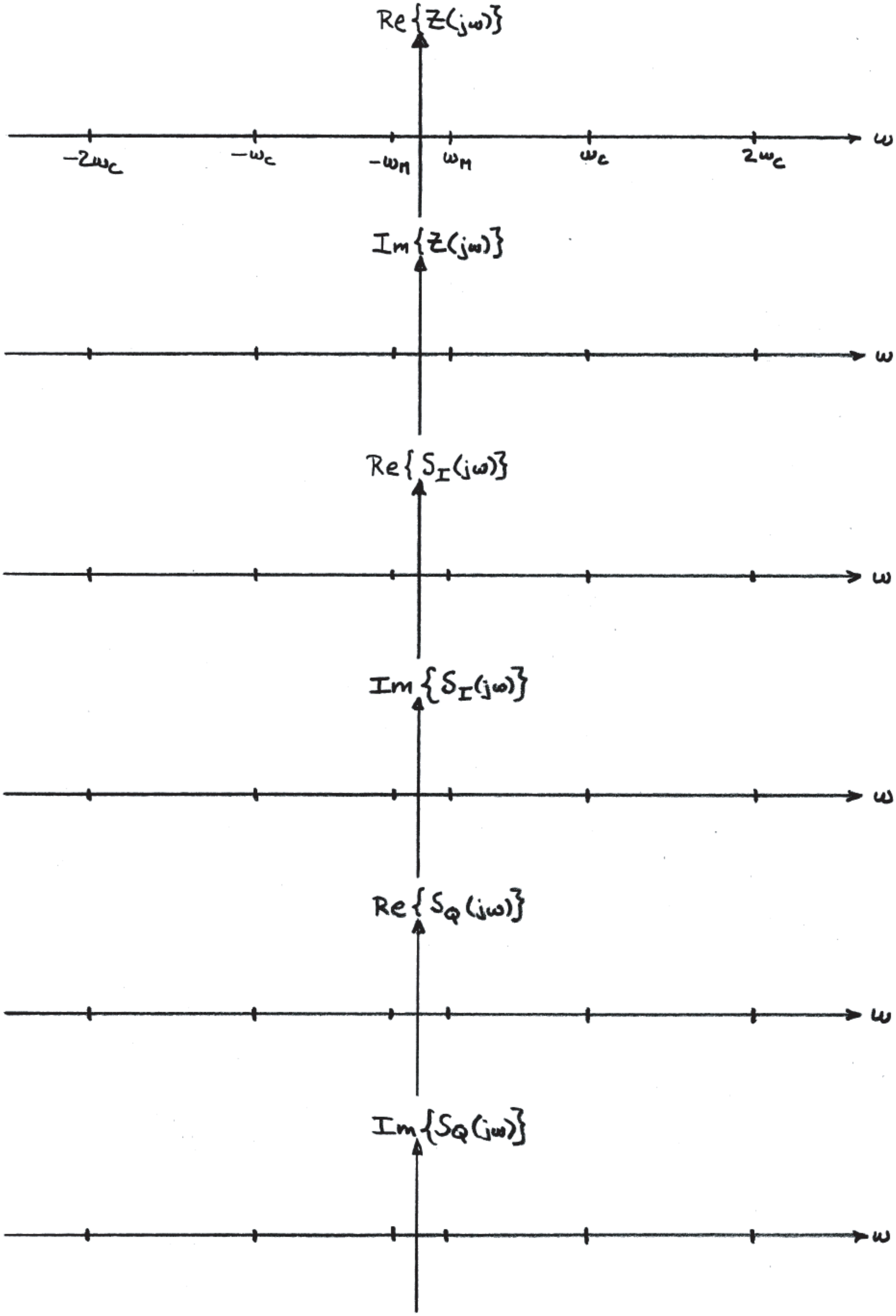
For part (d), the channel filter must be considered.

(d) [10 pts.] An important consideration in the design of a quadrature modulator/demodulator is the mitigation of gain and phase imbalance between the in-phase and quadrature branches. These can cause *crosstalk*, i.e., the appearance of some portion of the quadrature message $x_Q(t)$ in the output $y_I(t)$ of the in-phase branch and vice-versa. Show that cross-talk is avoided in the system above provided that the channel filter $H_c(j\omega)$ is conjugate symmetric about the carrier frequency ω_c in the sense that

$$H_{LP}(j\omega)\left[H_c^*(j(\omega_c - \omega)) - H_c(j(\omega_c + \omega))\right] = 0.$$

It is enough to only consider the in-phase branch.

Axes for plots found on next page.



Re{Z(jω)}
−2ωc
−ωc
−ωM
ωM
ωc
2ωc
ω
Im{Z(jω)}
ω
Re{SI(jω)}
ω
Im{SI(jω)}
ω
Re{SQ(jω)}
ω
Im{SQ(jω)}
ω