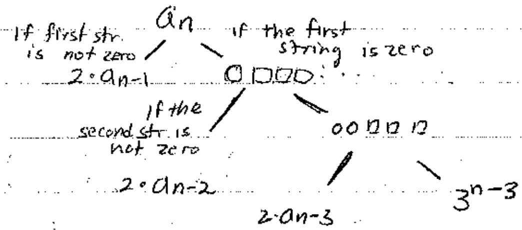
SELECTED PROBLEMS FROM the REVIEW.

(1) Let a_n = # of ternary string containing 3 or more zeros.



$$a_n = 2(a_{n-1} + a_{n-2} + a_{n-3}) + 3^{n-3}$$

init. cond $a_1 = 0$

$a_2 = 0$

$a_3 = 1$

(2) Solve $a_n = 5a_{n-1} - 4a_{n-2} + 3 \cdot 2^n$

$a_0 = 1 \quad a_1 = 10$

(1) Homogeneous Soln.

$$\chi(\lambda) = \lambda^2 - 5\lambda + 4 = (\lambda - 1)(\lambda - 4)$$

$$\lambda = 1, 4$$

$$a_n^h = 1^n(b_0) + 4^n(b_0) = \alpha_1 + \alpha_2 4^n$$

(2) Particular Soln.

$$F(n) = 3 \cdot 2^n$$

Recall: Given $s = S^n$, t = poly deg t m = mult. s in $\chi(\lambda) = 0$

$$a_n^p = S^n \cdot \text{poly deg } t \cdot n^m$$

thus,

$$a_n^p = 2^n \cdot c_0 \cdot n^0 = c_0 2^n$$

(3) Sum.

$$a_n = a_n^h + a_n^p = \alpha_1 + \alpha_2 4^n + \alpha_3 2^n$$

(4) Init Cond.

$$a_0 = 1 = \alpha_1 + \alpha_2 + 2\alpha_3 \ \ldots \ \text{solve!}$$

★ (#3) for $n \geq 2$ ← do not neglect

$$p = \frac{\binom{98}{n-2}}{\binom{100}{n}}$$

(#4) G is strongly directed G. Prove existence of path going through all vert

Prf Induct on n

$n=1$ yes

Assume $n>1$ and for any strongly connected graph w/ vertices $\leq n-1$ there exist a path that passes through all vertices.

Pick any vertex $\in G$ and consider $G' = G \setminus \{t_0\}$.

(note that G' may not be strongly connected).

Write

$G' = \bigcup_{i=1}^{k} G_i$ as the disjoint union of its strongly connected components.

Then, G' is strongly connected. By inductive hypothesis, $\exists$ direct path P_i on G_i.

(#5) If $f_n = f_{n-1} + f_{n-2}$ ... $f_1 = f_2$ ⌉ fibonacci

$f_1 + f_2 + \dots + f_m = f_{m+2} - 1$

inspiration(?)

$$= -f_2 + (\underbrace{f_1 + f_2}_{f_3} + \underbrace{f_2 + f_3}_{f_4} + \underbrace{f_4 + f_5}_{f_6})$$

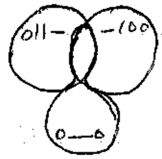
$$-f_2 + f_5 + f_6$$

$$-f_2 + f_7$$

$$= f_7 - 1$$

(#6) Binary string w/ particular input

think



$$2 \cdot 2^{n-3} + 2^{n-2} - 2 \cdot 2^{n-4} + 2^{n-6}$$

(#8) Multinomial...

$$\binom{10}{5}\binom{5}{3}\binom{2}{2} = \frac{10!}{5!3!2!}$$

(#10) Expected Val. Prblm.

$$P(\text{Roll } 5 \text{ zero times}) = \setminus \{(1,4), (4,1), (2,3)(3,2)\} = \frac{4}{36} = \frac{1}{9}$$

$$= \left(1 - \tfrac{1}{9}\right)^{20}$$

can roll 5 @ any of 20 rolls

$$P(\text{Roll } 5 \text{ exactly } 1) = 20 \cdot \left(\tfrac{1}{9}\right)^1 \left(1 - \tfrac{1}{9}\right)^{19}$$

$$P(\text{Roll } 5 \text{ exactly } 2) = \binom{20}{2}\left(\tfrac{1}{9}\right)^2\left(1 - \tfrac{1}{9}\right)^{18}$$

~~//~~ $3 = \binom{20}{3}\left(\tfrac{1}{9}\right)^3\left(1 - \tfrac{2}{9}\right)^{17}$

$P(\text{roll } 5 < 3) \approx 0.6134$

$P(\text{roll } 5 \geq 3) \approx 0.3865$

$E(x) = (0.6134)(-4) + (3)(.3865) < 0$ ✓ Don't play

(#11) SLOT MACHINE SHOWS 1, 2, 3

$P(1) = 0.1$
$P(2) = 0.3$
$P(3) = 0.6.$

consider one play/pull as showing a single #. Valid since ~~we ne~~ each num is independently chosen.

$$E(G) = (0.1)1 + (0.3)2 + (0.3)3$$

$= ~~va~~ \; 0$ / pull.

(#12) Chessboard — Recall.

$$(1-x)(1 + x + x^2 + \dots + x^k) = (1 - x^{k+1})$$

$$\sum_{n=0}^{k} x^n = \frac{1-x^{k+1}}{1-x}$$

(#13) Use induction to show

$$\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\cdots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$$

Base case: $n=2$; $1-\frac{1}{4} = \frac{2+1}{2(2)} = \frac{3}{4}$ ✓

Inductive step: Suppose $P(n-1)$ is true. Then,

$$\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\cdots\left(1-\frac{1}{(n-1)^2}\right)\left(1-\frac{1}{n^2}\right)$$

$$= \frac{n \cancel{+1}}{2(n-1)}\left(1-\frac{1}{n^2}\right)$$

$$= \frac{n \cancel{+1}}{2(n-1)} - \frac{n \cancel{+1}}{2(n-1)n^2} = \cancel{\frac{n^2(n+1) - n+1}{2n(n^2)}}$$

$$= \frac{n}{2(n-1)} - \frac{1}{(2(n-1)n)} = \frac{n^2-1}{2(n-1)(n)} = \frac{(n+1)(n-1)}{2(n-1)n}$$

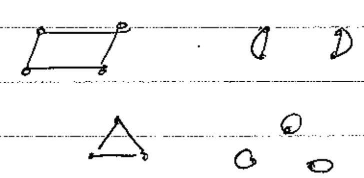
$$= \frac{n+1}{2n}$$ ✓

(#14) Soln

Trick Q: deg seq must add up to odd by the handshak lemma

$5+5+3+2+1+3 = 19$

(#15) Same deg seq but not isomorphic.



(#16) If there is a row / column of zeros.

(#17) G is simple w/ $> N = \frac{(n-1)(n-2)}{2}$ edges.

Show G is connected.

Soln

Suppose NOT. Let n vert have $(n-1)(n-2)/2$ disconnected G. Then

$\frac{(n-1)(n-2)}{2} = \binom{n-1}{2}$ subgraphs.

# of edges $G \le \underbrace{\binom{n-k}{2} + \binom{k}{2}}_{\text{disconnected subgraphs}} \le \binom{n-1}{2}$