LTI system properties

Property 8 – Stability

A Discrete Time (DT) or Continuous Time (CT) signal is said to be stable if and only if

$$\sum_{k=-\infty}^{\infty} |h[k]| \; or \int_{-\infty}^{\infty} |h(\tau)| d\tau \; is\; finite$$

Proof

Given that x(t) is a CT signal input to the LTI system, where the magnitude of x(t), |x(t)| is bounded by a constant, β for all values of t.

$$|x(t)| < \beta \; \forall \; t$$

Then, using the commutative property for convolution,

$$y(t) = x(t) * h(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$\therefore |y(t)| = \left| \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \right| = \int_{-\infty}^{\infty} |h(\tau)||x(t-\tau)|d\tau \; \leq \; \int_{-\infty}^{\infty} |h(\tau)|\beta d\tau$$

The proof goes parallel for the DT case.

*In other words, if |h(t)| or |h[n]| diverges to infinity for any t or n, then the system is not stable.

E.g.

$h_1(t) = e^{-t} u(t)$

$h_1(t)$ is stable for all values of t, where when $t < 0$, $h_1(t) = 0$, and for $t \geq 0$, $h_1(t) \leq 1$

$h_2(t) = e^{t} u(t)$

$h_2(t)$ is not stable because for $t > 0$, e^t goes to infinity as t increases.