Filters

Thursday, September 20, 2007
3:30 PM

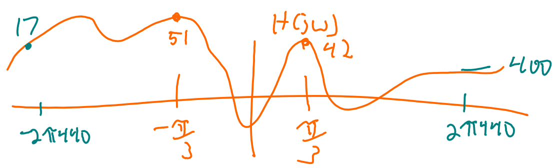
$$e^{st} \rightarrow \boxed{} \rightarrow H(s)\, e^{st}$$

$$H(s) = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt$$ – Frequency response

$$e^{j\pi t} \rightarrow \boxed{} \rightarrow H(j\pi)\, e^{j\pi t}$$

$$e^{j\frac{\pi}{3}t} \rightarrow \boxed{} \rightarrow H(j\tfrac{\pi}{3})\, e^{j\frac{\pi}{3}t}$$

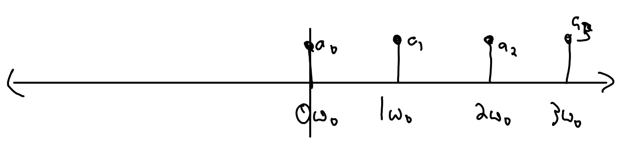
$$e^{j\omega t} \rightarrow \boxed{} \rightarrow H(j\omega)\, e^{j\omega t}$$



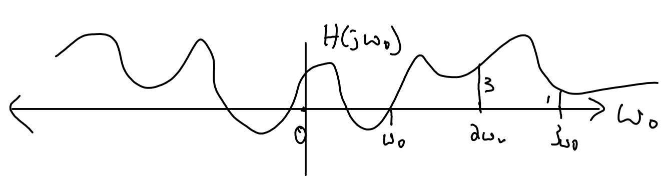
$$e^{j\frac{\pi}{3}t} + e^{-j\frac{\pi}{3}t} \rightarrow \boxed{} \rightarrow H(j\tfrac{\pi}{3})\, e^{j\frac{\pi}{3}t} + H(-j\tfrac{\pi}{3})\, e^{-j\frac{\pi}{3}t} =$$
$$= 51\, e^{j\frac{\pi}{3}t} + 42\, e^{-j\frac{\pi}{3}t}$$

$$x(t) = \cos(2\pi 440 t) \rightarrow \boxed{} \rightarrow ?$$

$$x(t) = \tfrac{1}{2}\left(e^{j2\pi 440 t} + e^{-j2\pi 440 t}\right) \rightarrow \boxed{} \rightarrow \overset{a_1}{\tfrac{1}{2}} H(j2\pi 440)\, e^{j2\pi 440 t} +$$
$$\overset{a_1}{\tfrac{1}{2}} H(-j2\pi 440)\, e^{-j2\pi 440 t} =$$
$$= \underset{\bar{a}_1}{\tfrac{1}{2}(400)}\, e^{j2\pi 440 t} + \underset{\bar{a}_1}{\tfrac{1}{2}(17)}\, e^{-j2\pi 440 t}$$



– plot of a_k's of $x(t)$



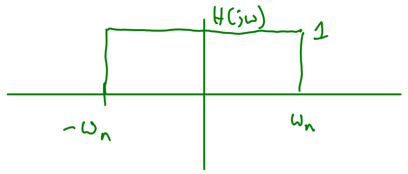
new

$$\bar{a}_k = H(jk\omega_0) \cdot a_k$$

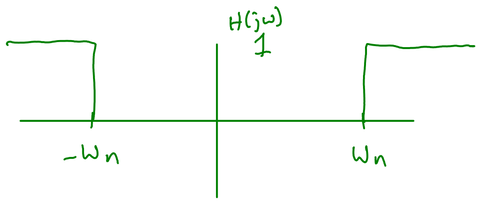
The LTI system only modifies the a_k values not the frequencies.

Observation: if a system takes an A 440 and transforms it into a C then the system can not be LTI

Typical examples of frequency response ($H(j\omega)$)



- low pass filter



~ High Pass

Question to summarize what you need to know

An LTI System has unit impulse response $h(t) = e^{-t}u(t)$

a. what is the output of the system when the input is $x(t) = u(t-3)$

– convolution

b. what is the frequency response of the system?

– Fourier

c. what is the system's response to $x(t) = \cos(3\pi t) + \left(\frac{1-j}{2}\right)\sin(5\pi t)$

Answers:

a) response $= x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$

$$= \int_{-\infty}^{\infty} u(\tau-3)\, e^{-(t-\tau)} u(t-\tau)\,d\tau = \begin{cases} \int_3^t e^{\tau-t}\,d\tau & t \geq 3 \\ 0 & t < 3 \end{cases}$$

$$= \left(e^{-t}\int_3^t e^{\tau}\,d\tau\right) u(t-3) = e^{-t}u(t-3)\left[e^{\tau}\right]_3^t = e^{-t}u(t-3)\left[e^{t} - e^{3}\right]$$

$$= \left(e^{-t}\int_3^t e^{\tau}\,d\tau\right) u(t-3) = e^{-t}u(t-3)\left[e^{\tau}\right]_3^t = e^{-t}u(t-3)\left[e^t - e^3\right]$$

$$= \left(e^{-t}\cdot e^t - e^3\cdot e^{-t}\right)u(t-3) = \left[1 - e^{3-t}\right]u(t-3)$$

b) $H(s) = \int_{-\infty}^{\infty} h(t)\,e^{-st}\,dt$ ← Laplase transform

c) use part b, write $x(t)$ as fourier series from part B

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \rightarrow \boxed{} \rightarrow \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0)\, e^{jk\omega_0 t}$$