Due Sept. 30 2009 at 5:00pm

ECE 438 Assignment No. 4 Fall 2009

1. Let $x[n]$ be a digital signal with DTFT given by

$$X(\omega) = \begin{cases} 1 - |\omega|/(\pi/2), & |\omega| \le \pi/2 \\ 0, & \pi/2 \le |\omega| \le \pi \end{cases} .$$

a. Carefully sketch $X(\omega)$.

Suppose that $y[n]$ is obtained by downsampling $x[n]$ by a factor of 3, i.e. $y[n] = x[3n]$.

b. Based on the DTFT relationship for downsampling that was derived in class, carefully sketch $Y(\omega)$.

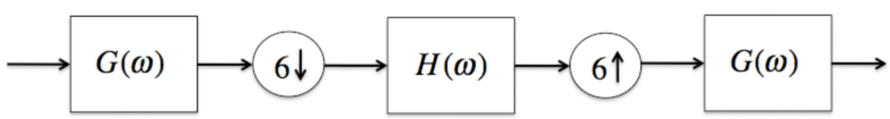
2. Consider the digital filter described by the following difference equation

$$y[n] = \left(x[n] + x[n-1] + x[n-2]\right)/3$$

a. Find a simple expression for the frequency response $H(\omega)$ of this filter.

b. Sketch the magnitude of $H(\omega)$.

Now consider the following digital system,



where $H(\omega)$ is the filter from parts a and b and $G(\omega)$ is an ideal low-pass filter with a cutoff frequency of $\pi/6$ rad/sample and unity gain in the passband.

c. Find the overall frequency response $F(\omega)$ for this system.

d. Sketch the magnitude of $F(\omega)$.

e. Discuss the possible advantages of a system like that shown above compared to directly implementing a digital filter with frequency response $F(\omega)$ as a single stage.

3. Conversion between analog and digital frequencies

a. Electrocardiogram signals are very susceptible to interference from the 60 Hz power present in the room where the patient is being monitored. You are going to design a high-pass digital filter to eliminate the 60 Hz interference and everything at frequencies below 60 Hz. Assume that the highest frequencies of interest in the electrocardiogram signal are at 2500 Hz. Choose an appropriate sampling frequency for your A/D convertor, and sketch the desired frequency response of the digital filter. Be sure to show how you calculated the cutoff frequency for the digital filter.

b. Long term climate change is a topic of great interest at this time. To see if there has been a significant long-term trend in temperatures in Lafayette, IN, you have downloaded temperature data from the U.S. Weather Service. The file contains the average monthly temperature at the Purdue airport for the past 100 years. Thus it consists of 1200 samples. In order to see if there is a long-term trend, you will need to remove the annual cycle from the data. Sketch the desired frequency response of an ideal low-pass digital filter that will accomplish this. Be sure to show how you calculated the cutoff frequency of the digital filter.

4. The signal $x(t) = 0.8\cos[2\pi(500)t] + 0.3\cos[2\pi(3500)t]$ is sampled at 10 kHz using an ideal A/D converter to produce the digital signal $x[n]$. You compute a 512-point DFT $X[k]$ of this signal. Find the approximate values of k and the amplitudes $|X[k]|$ corresponding to the spectral peaks in the analog signal.

5. Consider the length N signal x[n] $\neq 0$, only for n=0, 1, …, N-1. Let y[n] be a new signal defined by padding x[n] with zeros to make it length M=LN for some integer L:

$$y[n] = \begin{cases} x[n], & n = 0,\ldots,N-1 \\ 0, & n = N,\ldots,M-1 \end{cases}$$

Let $X(\omega)$ be the DTFT of x[n] and let Y[k], k=0,…, M-1 be the M point DFT of y[n].

Show that Y[k] yields samples from $X(\omega)$ at points $\omega_k = 2\pi k / M$, k=0,1,…, M-1, i.e.

$$Y[k] = X\left(\frac{2\pi k}{M}\right), k = 0,\ldots,M-1 .$$