2) Graphical method of Convolution

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

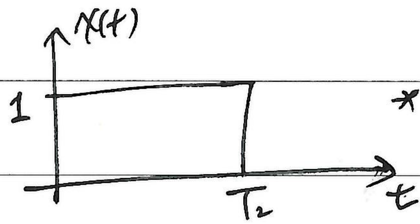
1. View both $x(\tau)$ and $h(t-\tau)$ as func. of τ.
2. $h(t-\tau) = h(-(\tau-t))$
   - flip $h(\tau)$ about $\tau=0$
   - shift to the right by t
3. point-wise multiply $x(\tau)\, h(-(\tau-t))$
4. Find area under product for all τ

* Remember that conv. is commutative

$\rightarrow$ can flip either $x(t)$ or $h(t)$ and obtain same answer.

Convolution Example

e.g. 1) $x(t) * h(t) = y(t)$



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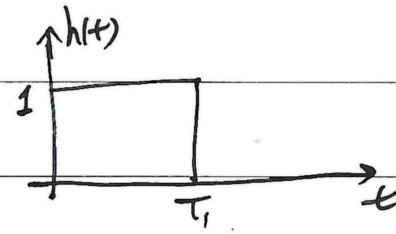
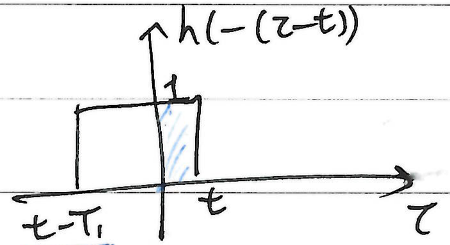
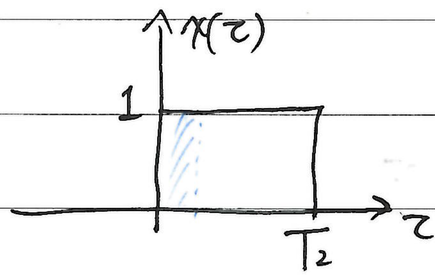
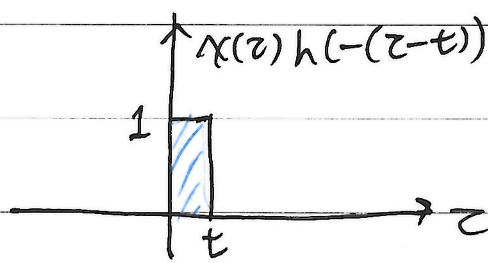


$= ?$





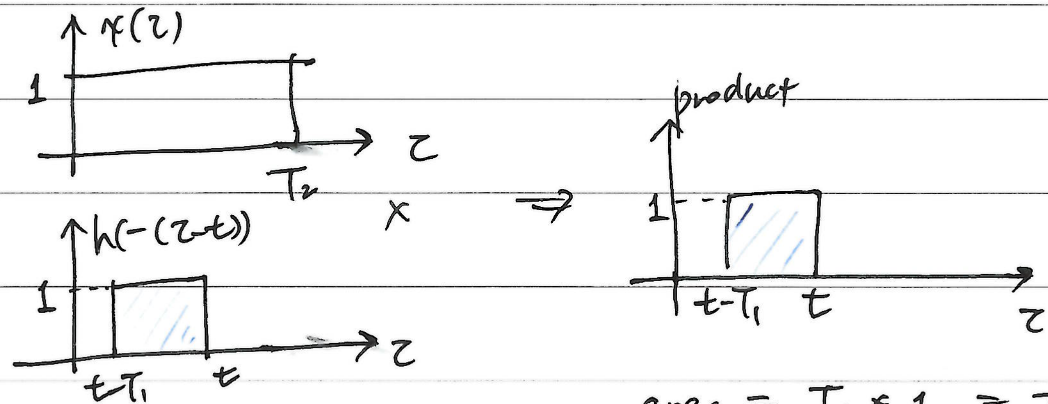


$\text{area} = t \times 1 = t$

- If $t<0$, "no overlap" $\Rightarrow y(t)=0$
- If $0<t<T_1$ $(t-T_1<0)$, "partial overlap"

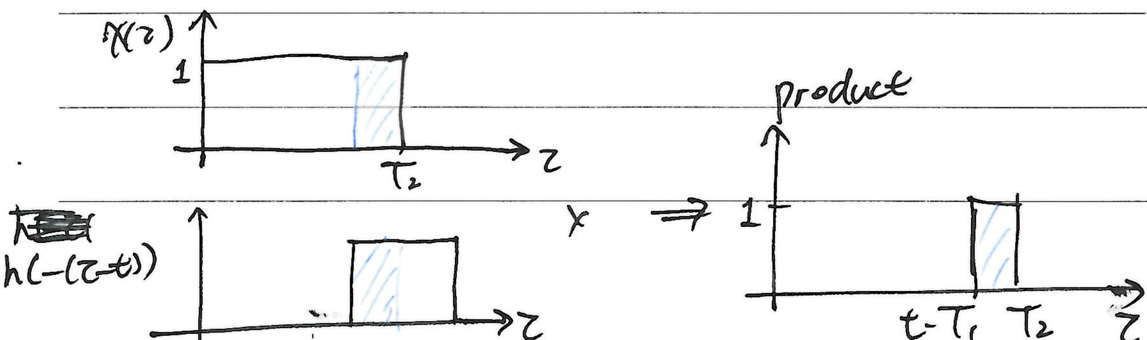
  $\Rightarrow y(t)=t$

- If $\begin{pmatrix} t-T_1>0 \\ t<T_2 \end{pmatrix} \Leftrightarrow T_1<t<T_2$ "full overlap"



area $= T_1 \times 1 = T_1$ $\Rightarrow y(t)=T_1$

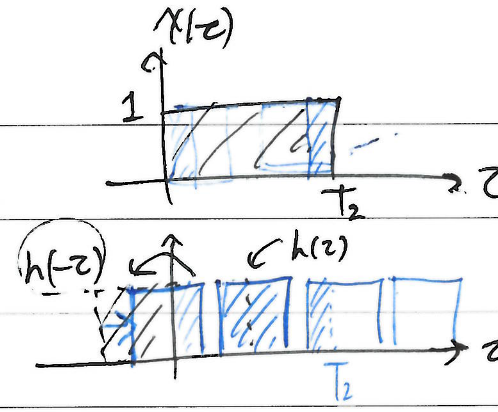
- If $\begin{pmatrix} t>T_2 \\ t-T_1<T_2 \end{pmatrix} \Leftrightarrow T_2<t<T_1+T_2$ "partial overlap"



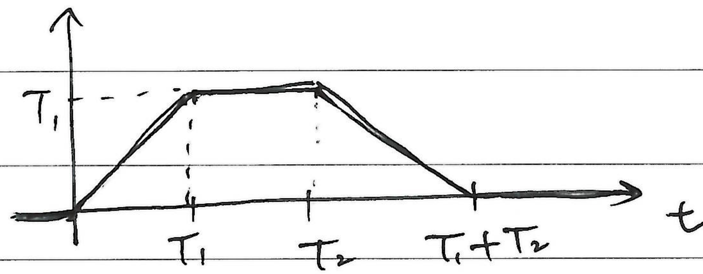
area $= \{T_2-(t-T_1)\}\times 1 = T_1+T_2-t \Rightarrow y(t)$

- Summarizing:

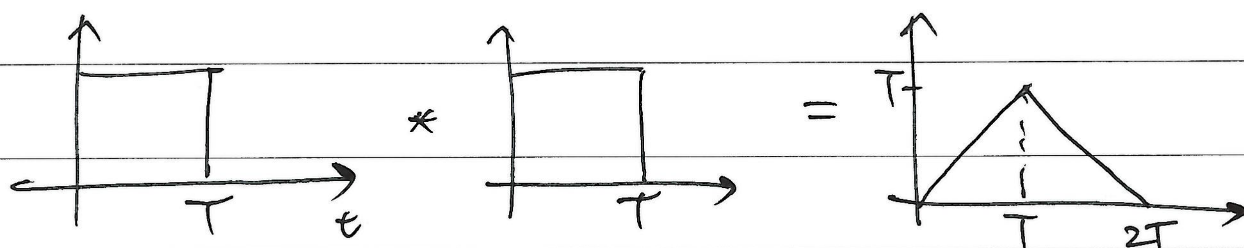
$$y(t) = \begin{cases} 0, & t<0 \\ t, & 0<t<T_1 \\ T_1, & T_1<t<T_2 \\ T_1+T_2-t, & T_2<t<T_1+T_2 \\ 0, & t>T_1+T_2 \end{cases}$$



$h(-(\tau-t))$

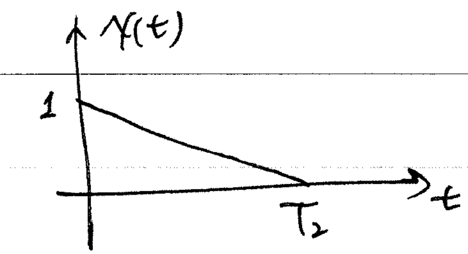


- ① Special case when $T_1 = T_2$ $(=T)$



- General principle: convolving $x_1(t)$ of duration T_1 with $x_2(t)$ of duration T_2 yields a result of duration $T_1 + T_2$

e.g.2) "Ramp-Down Triangle"



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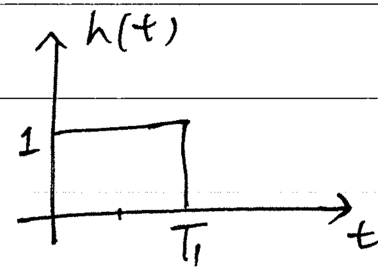
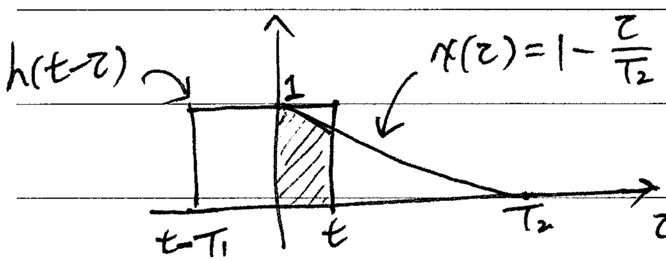


$= y(t) = ?$

$$x(t) \longrightarrow \boxed{\begin{matrix} \text{LTI} \\ h(t) = \mathrm{rect}\left(\frac{t - T_1/2}{T_1}\right) \end{matrix}} \longrightarrow y(t)$$

$$y(t) = \int_{t-T_1}^{t} x(\tau)\, d\tau \qquad (\text{assume } T_2 > T_1)$$

- for $t < 0$, $y(t) = 0$ (no overlap)
- for $\begin{pmatrix} t > 0 \\ t - T_1 < 0 \end{pmatrix} \Leftrightarrow 0 < t < T_1$ (partial overlap)



$$y(t) = \int_0^t \left(1 - \frac{\tau}{T_2}\right) d\tau = t - \frac{1}{T_2}\frac{\tau^2}{2}\Big|_0^t$$

$$= (t - 0) - \frac{1}{2T_2}(t^2 - 0^2)$$

$$= -\frac{1}{2T_2} t^2 + t \qquad (\text{concave downwards})$$