ECE 495N EXAM I
Friday, Oct.2, 2009 GRIS 280, 1130A-1220P

CLOSED BOOK :

The following equations will be provided in the exam.

$$I = \frac{2q}{\hbar}\int_{-\infty}^{+\infty} dE\ D(E-U)\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\left[f_1(E)-f_2(E)\right] \quad (1)$$

with $$N = 2(for\ spin)\int_{-\infty}^{+\infty} dE\ D(E-U)\frac{\gamma_1 f_1(E)+\gamma_2 f_2(E)}{\gamma_1+\gamma_2} \quad (2)$$

and $$U = U_L + U_0(N-N_0) \quad (3)$$

Fermi functions : $$f_1(E) = \frac{1}{e^{(E-\mu_1)/kT}+1} \text{ and } f_2(E) = \frac{1}{e^{(E-\mu_2)/kT}+1} \quad (4)$$

Law of equilibrium : $$P_\alpha = \frac{1}{Z}\exp(-(E_\alpha - \mu N_\alpha)/kT) \quad (5)$$

The exam will have three questions (maximum score: 8+8+9 = 25) covering the three topics we have covered until Friday Sept.25 in class as described in the course outline:

	Pages
1 / An atomistic view of electrical resistance	1-18, 21-27
See also http://www.nanohub.org/courses/cqt, CQT Lecture 2 (80 mins)	
2 / Schrodinger equation	33-49
Hydrogen atom, Method of finite differences	
3 / Self-consistent field / Coulomb blockade	18-20, 51-78
One-electron versus the many-electron picture	
See also http://www.nanohub.org/courses/cqt, CQT Lecture 4 (70 mins)	

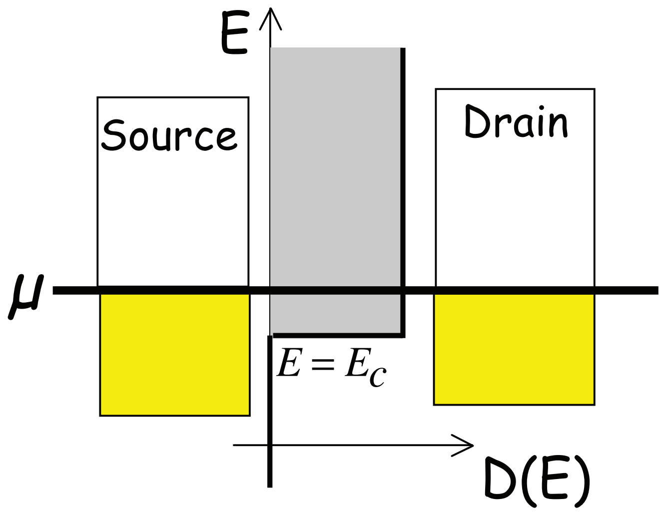
All page numbers refer to the recommended reference #1: S. Datta, Quantum Transport: Atom to Transistor, Cambridge University Press (2005), ISBN 0-521-63145-9.

Non-MATLAB HW questions are a good guide to the kinds of questions on the test. In addition a few example questions are listed below to help you prepare. Solutions to these will also be posted.

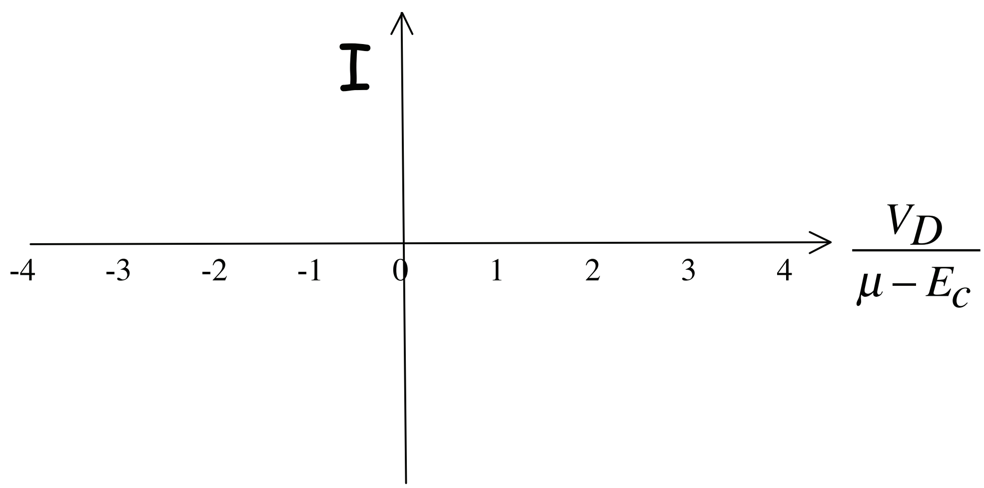
#**1.** Assume $U_0 = 0$ and the Laplace potential U_L to be a fraction α of the drain potential V_D (the source potential is assumed zero):

$U_L = -q\,\alpha V_D$, α being a constant between 0 and 1.

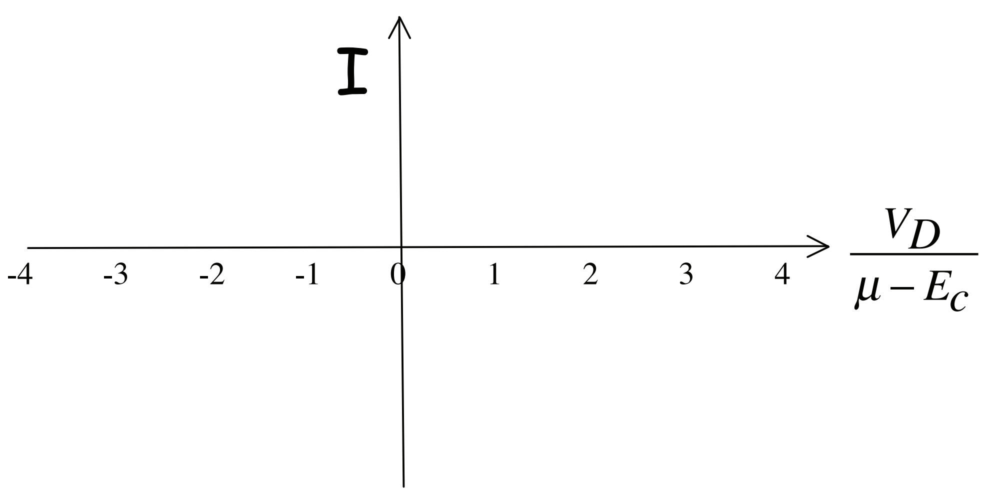
A channel has a density of states as shown, namely a constant non-zero value for $E \geq E_C$ and zero for $E < E_C$. Assume that the equilibrium electrochemical potential μ is located above E_C as shown. Sketch the current versus drain voltage assuming that the electrostatic potential of the channel (a) remains fixed with respect to the source ($\alpha = 0$) and (b) assumes a value halfway between the source and drain potentials ($\alpha = 0.5$),
Explain your reasoning clearly.



(a) Channel potential fixed with respect to source: $\alpha = 0$.



(b) Channel potential halfway between source and drain: $\alpha = 0.5$



#2. We have seen in class that free electrons in the absence of any external potential are described by (in one dimension)

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} \qquad (1)$$

whose solutions can be written in the form $\psi(x,t) = \underbrace{C}_{constant}\, e^{+ikx}\, e^{-iEt/\hbar}$ (2)

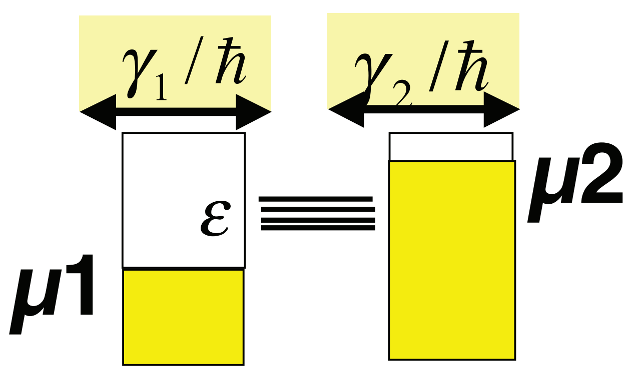
with E and k related by the dispersion relation: $E = \hbar^2 k^2 / 2m$ (3)

We have also seen that if the electrons are confined in a box of length L, the energy levels become discrete with the lowest energy given by $E_1 = \hbar^2 \pi^2 / 2mL^2$ (4)

(a) Can you suggest a suitable differential equation to replace (1) if you wanted the dispersion relation to look like $E = A k^4$ (3')
(A being a constant) instead of (3) ?

(b) If a system of electrons with a dispersion relation given by (3') were confined in a box of length L, how would the expression for the lowest energy given in (4) be modified?

#3.



A box has four degenerate energy levels all having energy ε. We know that for non-interacting electrons the maximum current under bias is $I = \frac{q}{\hbar}\frac{4\gamma_1\gamma_2}{\gamma_1+\gamma_2}$

Use the multielectron picture to
(a) derive the correct expression for the maximum current if the electron-electron interaction energy is so high that no more than one electron can be inside the box at the same time,

(b) find the average number of electrons in the channel if it is in equilibrium with chemical potential μ and temperature T? Your answer should be in terms of ε, μ and T.

#4. A channel has four degenerate energy levels all having the same energy $\varepsilon = 0$ eV with an interaction energy that can be written as $U_{ee} = U_0 N(N-1)/2$, where $U_0 = 0.1$ eV. The figure below shows the change in the ***equilibrium*** number of electrons, N inside the channel as the electrochemical potential μ is changed. What are the values of μ at which the transitions in N take place (labeled $\mu 1, \mu 2, \mu 3$ and $\mu 4$ in the figure) ?