Know this!

Error

$\frac{\pi}{4} \int_0^1 \sqrt{1-x^2}\, dx$

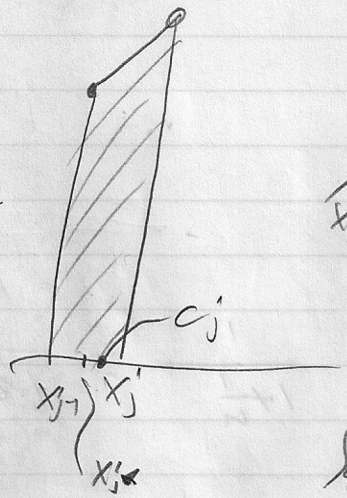
How big should N be?

Prob = How many subdivisions do we get w/i 1000th of π

$$I = \int_a^b f(x)\,dx = \lim_{N\to\infty} \sum_{j=1}^{N} f(x_j')\,\Delta x \qquad \Delta x = \frac{b-a}{N}$$

$$\text{Error} = I - \sum_{j=1}^{N} f(x_j')\,\Delta x$$

MVT's



$$\int_{x_{j-1}}^{x_j'} f(x)\,dx = f(x_j^*)\underbrace{(x_j' - x_{j-1})}_{\Delta x}$$

$\int_a^b f(x)\,dx$ = sum of those $= \sum_{j=1}^{N} f(x_j^*)\,\Delta x$

$$\text{Error} = \underbrace{\int_a^b f(x)\,dx}_{\sum_1^N f(x_j^*)\Delta x} - \sum_1^N f(x_j)\,\Delta x = \sum_1^N \underbrace{\left(f(x_j^*) - f(x_j)\right)}_{f'(c_j)\Delta x}\Delta$$

$$|E| \le \sum_{j=1}^{N} |f'(c_j)|\Delta x^2$$

Get $|E| \le \underbrace{\sum_{j=1}^{N} M(\Delta x)^2}_{NM(\Delta x)^2}$

now suppose $M = \max_{[a,b]} |f'|$

$$|E| \le NM\left(\frac{b-a}{N}\right)^2$$

$$|E| \le \frac{M(b-a)^2}{N}$$