(10 pts) **1.** State the sampling theorem. (You may use your own words but be precise!)

7

A band-limited signal can be recovered ~~by~~ from sampling if the sampling frequency w_s is greater than $2w_m$, where w_m is the ~~cut off~~ frequency of what?

$$T = \frac{2\pi}{w_s}$$

If a signal $x(t)$ is such that $X(w)=0$ when $|w| > w_m$. ~~for~~ some w_m, then $x(t)$ can be recovered from the sampling $x_d[n] = x(nT)$ whenever $\frac{2\pi}{T} > 2w_m$

2. Consider the signal $x(t) = \cos(3\pi t) + \cos(5\pi t)$.

(5 pts) a) Is $x(t)$ band-limited? If so, what is the Nyquist rate for this signal (Justify your answers.)

5

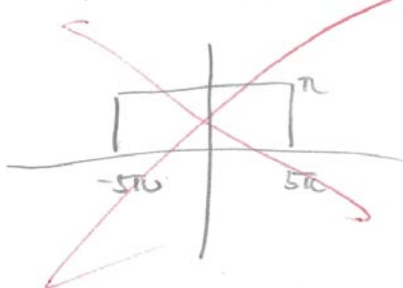
$$x(t) = \frac{1}{2}e^{j3\pi t} + \frac{1}{2}e^{-j3\pi t} + \frac{1}{2}e^{j5\pi t} + \frac{1}{2}e^{-j5\pi t}$$

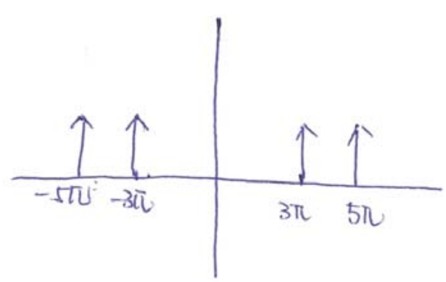
By 17:

$$X(w) = \frac{1}{2}\cdot 2\pi\,\delta(w-3\pi) + \frac{1}{2}\pi\delta(w+3\pi) + \frac{1}{2}\pi\,\delta(w+5\pi) + \frac{1}{2}\pi\,\delta(w-5\pi)$$

$$= \pi\left(\delta(w-3\pi) + \delta(w+3\pi) + \delta(w-5\pi) + \delta(w+5\pi)\right)$$

Yes, $w_N = 2\cdot 5\pi = 10\pi$





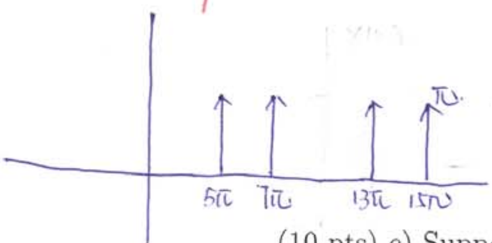
(10 pts) b) Suppose the signal $x(t)$ is modulated by the carrier signal $c(t) = e^{10j\pi t}$. Sketch the graph of the modulated signal in the frequency domain. Can one recover $x(t)$ from the modulated signal? If you answered yes, draw the diagram of a system which could be used to demodulate the signal. If you answered no, explain why.

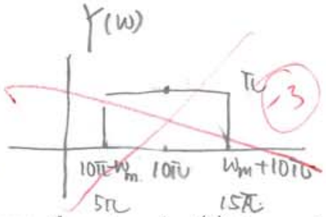
7

$y(t) = x(t)\, e^{10j\pi t}$

$Y(w) = X(w - 10\pi)$ by. 11.

Yes. $x(t)$ can be recovered





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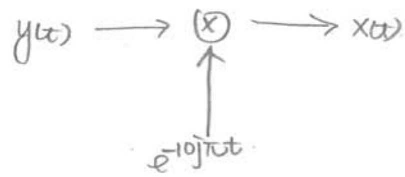


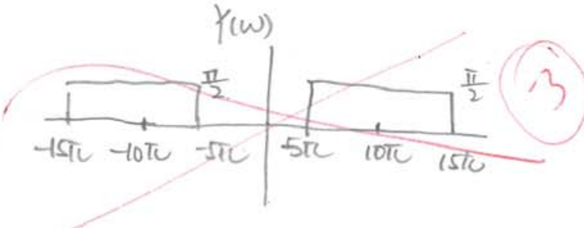
(10 pts) c) Suppose the signal $x(t)$ is modulated by the carrier signal $c(t) = \cos(10\pi t)$. Sketch the graph of the modulated signal in the frequency domain. Can one recover $x(t)$ from the modulated signal? If you answered yes, draw the diagram of a system which could be used to demodulate the signal. If you answered no, explain why.

6

$y(t) = x(t) \cos(10\pi t) = x(t)\, \frac{1}{2}\left(e^{10\pi j t} + e^{-10\pi j t}\right)$

$= \frac{1}{2}\left(x(t) e^{10\pi j t} + x(t) e^{-10\pi j t}\right)$

$Y(w) = \frac{1}{2}\left(X(w - 10\pi) + X(w + 10\pi)\right)$



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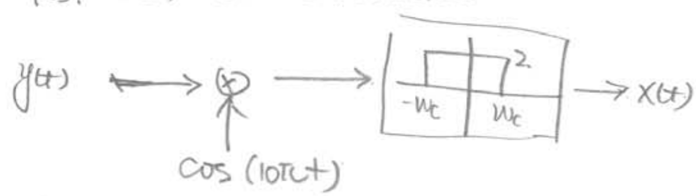
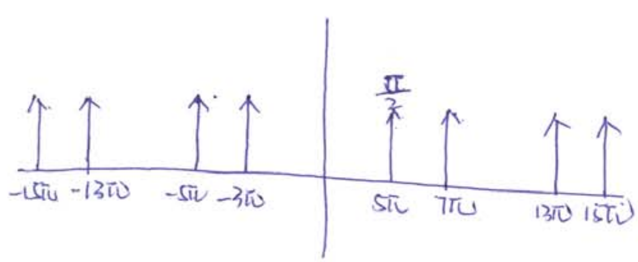
Yes. $x(t)$ can be recovered

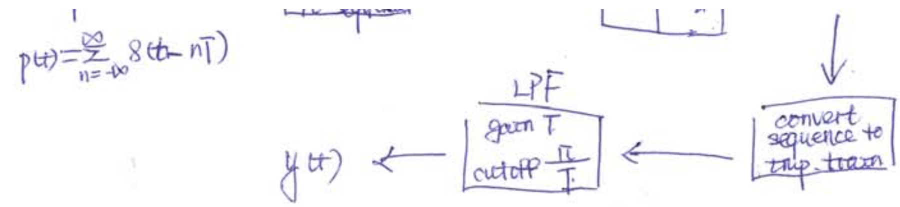


$w_m < w_c < w_m + 10\pi$

namely?

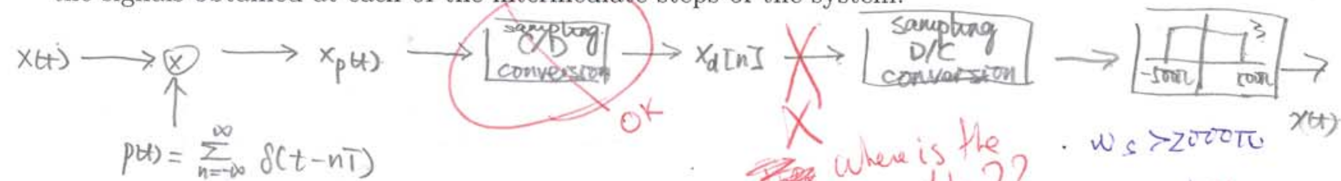
-1





9

(20 pts) **3. DT processing of CT signals.** A CT signal $x(t)$ with Nyquist rate equal to 2000π is the input of a CT system consisting of a low-pass filter with gain 3 and cut-off frequency 500π. Draw the diagram of an *equivalent* system which would sample $x(t)$, process the samples using a DT system, and reconstruct a CT signal from the processed samples. (Don't forget to specify an appropriate frequency for the sampling as well as the frequency response of the DT system.) Illustrate all the steps of the system by sketching (in the frequency domain) the graph of an input signal together with the graphs of the signals obtained at each of the intermediate steps of the system.

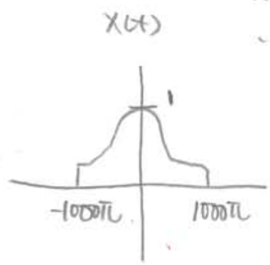


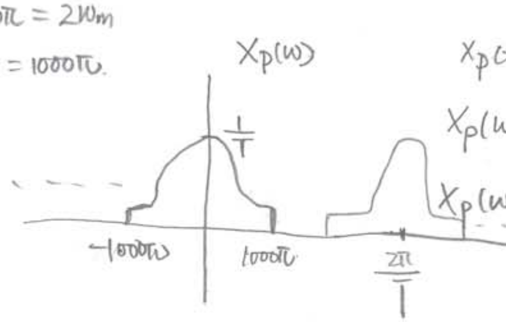
$\cdot\ \omega_s > 2000\pi$

$T < \frac{1}{1000}$ 3/10

$\omega_N = 2000\pi = 2\omega_m$

$\therefore \omega_m = 1000\pi$





$x_p(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT)$

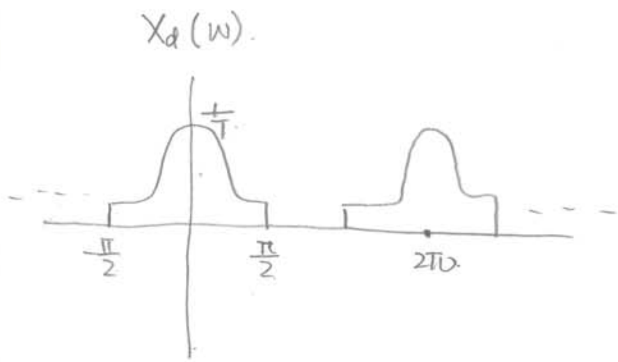
$X_p(\omega) = \frac{1}{2\pi}X(\omega) * F\left(\sum_{n=-\infty}^{\infty}\delta(t-nT)\right)$ by 14

$X_p(\omega) = \frac{1}{2\pi}X(\omega) * \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$ by 22

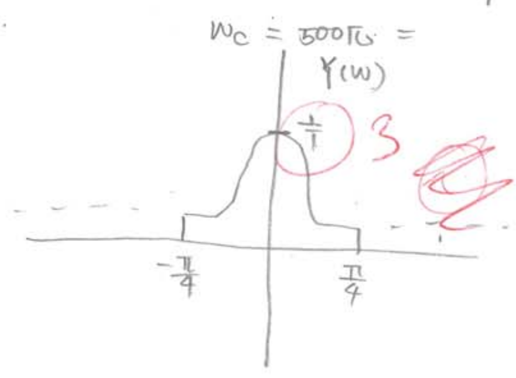
$= \frac{1}{T}X(\omega) * \sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$.

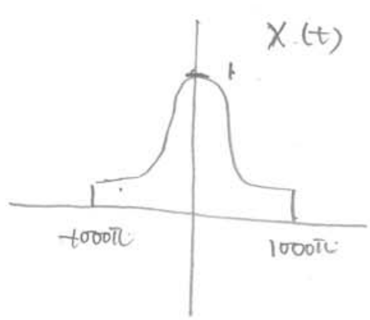
$T = \frac{2\pi}{\omega_c} = \frac{2\pi}{500\pi} = \frac{1}{250}$

-1



$\omega_c = 500\pi =$





6/10

10

(15 pts) **4.** Compute the Laplace transform of the following signal *without using the table of Laplace transform pairs*:

$$x(t) = e^{-2t}u(t) + e^{5t}u(-t).$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} \left(e^{-2t} u(t) + e^{5t} u(-t) \right) e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-(2+s)t} u(t)\, dt + \int_{-\infty}^{\infty} e^{(5-s)t} u(-t)\, dt$$

$$= \underbrace{\int_{0}^{\infty} e^{-(2+s)t}\, dt}_{\text{if } Re(s) > -2} + \underbrace{\int_{-\infty}^{0} e^{(5-s)t}\, dt}_{\text{if } Re(s) < 5}$$

$$= \left. \frac{e^{-(2+s)t}}{-(2+s)} \right|_{0}^{\infty} + \left. \frac{e^{(5-s)t}}{5-s} \right|_{-\infty}^{0}$$

$$= \left(0 - \frac{1}{-(2+s)} \right) + \left(\frac{1}{5-s} - 0 \right)$$

$$= \frac{1}{2+s} + \frac{1}{5-s}$$

ROC ?

ROC : $-2 < s < 5$