If you would like to time yourself and make this a 2 hour exam, do one of problems 1 and 2, do one of problems 3 and 4, and do all of problems 5-8.

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1. Let $A = \{(x, y) \in \mathbb{R}^2; a < x < b, c < y < d\}$ and show A is open in $\mathbb{R}^2$. Of course we assume $0 < a < b$ and $0 < c < d$.

2. Prove that closed subsets of compact sets are themselves compact.

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3. Let (X, d) be a metric space and A a non-empty subset. Show $x \in A'$ iff there exists a sequence $\{x_n\} \subseteq A$ so that $x_n \to x$ and $\forall n, x_n \neq x$.

4. Let (X, d) be a metric space and $\{x_n\}_{n\in\mathbb{N}}$ a sequence in X which converges to $x \in X$. Prove or disprove $\{x_n\}_{n\in\mathbb{N}} \cup \{x\}$ is compact.

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5. Show that a sequence in a metric space converges to a point x iff every subsequence has in turn a subsequence which converges to x.

6. Can a countable subset of a metric space be open? Prove or disprove (i.e. example or disproof).

7. Show that every nonempty connected open set in $\mathbb{R}$ is of the form (a, b) for $a \in [-\infty, \infty)$ and $b \in (-\infty, \infty]$.

8. Recall the definition of a perfect set; i.e. a set A, in a metric space (X, d) is perfect iff $A' = A$. Show A is perfect and nonempty implies A is uncountable.