Last time ...

There are two types of sample spaces:
discrete and continuous.

A <u>discrete</u> sample space has a finite or countable number of outcomes.

<u>Ex</u> Flip a coin n times and observe the number of heads.

Let K be the number of heads obtained in n flips, $0 \le K \le n$

$S = \{0, 1, \ldots, n\}$ $\Rightarrow$ S is discrete

↑ finite

<u>Ex</u> Flip a coin until we observe the first heads.

Let M be the number of flips needed until we observe the first heads, $1 \le M < \infty$

$S = \{1, 2, \ldots\}$ $\Rightarrow$ S is discrete

↑ countable

A continuous sample space has an ~~con~~ uncountable number of outcomes.

<u>Ex</u> throw a dart and measure the distance from the center of the dartboard to the dart. Let R be the distance.

$0 \le R < \infty$

$S = [0, \infty)$ => S is continuous

uncountable

Probability

A <u>probability</u> is a number assigned to an event that measures the likelihood of an event occurring

Will denote $Pr(A)$ as the probability of the event A.

Note: Sometimes see $P(A)$

Would like an ~~definition~~ assignment of probability that:

1) Useful in a mathematical model
2) Agrees with "notion" of likelihood (connection to real world)

Two approaches: relative frequency and axiomatic

Relative Frequency Approach

Let A be an event in a random experiment

The relative frequency $r_A(n)$ of event A is

$$r_A(n) = \frac{N_A}{n},$$

where N_A = # of times A occurs in n trials of the experiment

Relative frequency appraach defines

$$Pr(A) = \lim_{n \to \infty} r_A(n) \qquad (Pr(A) \approx r_A(n) \text{ for } n \text{ large})$$

Ex Consider an urn with 2 black balls and 3 white balls.
Would like to find the probability of each outcome.
$S = \{b, w\}$

Disadvantages:

1) May ~~fep~~ need to repeat many times before $Pr(A)$ converges $\rightarrow$ not feasible in some cases
2) N_A ~~is~~ may be different each time we perform n trials.

Axiomatic Approach

In the axiomatic approach probabilities are assigned to events such that they satisfy a set of axioms.

Need set theory

Review of Set Theory

Def: A set is a collection of objects called elements.

A set is always contained by some largest set S, which is a set containing all possible elements.

Note: S varies depending on the context

Notation: Sets are usually denoted by capital letters (A, B, C)

$a \in A$ means a is an element of A

$a \notin A$ means a is not an element of A

~~W~~

Ways of specifying sets:

1) List elements explicitly

e.g. $A = \{1, 2, 3\}$, $B = \{1, 2, \ldots, n\}$

$C = \{1, 3, 5, \ldots\}$

2) Specify a rule for membership

e.g. $A = \{x \in \overset{S}{\mathbb{R}} : x \geq 0\} = [0, \infty)$

(such that

$B = \{y \in \mathbb{R} : -2 < y \leq 15\} = (-2, 15]$

$C = \{z \in \mathbb{R} : f(z) = 2\}$, where f is a function

3) Interval notation

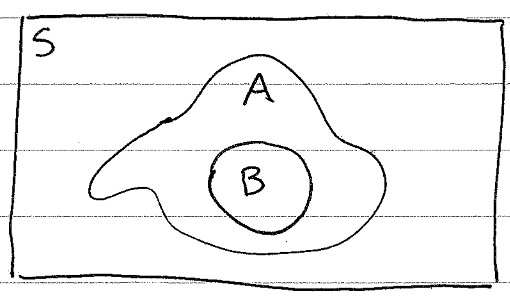
$A = (2, 3]$, $B = [5, 10)$

<u>Def:</u> Let A be a set. B is a subset of A if all the elements of B are also in A.

We write this as $B \subset A$

$(b \in B$ implies $b \in A)$

We can use Venn diagrams to describe sets and their relationships.



$B \subset A$

<u>Note:</u> Every set is a subset S.

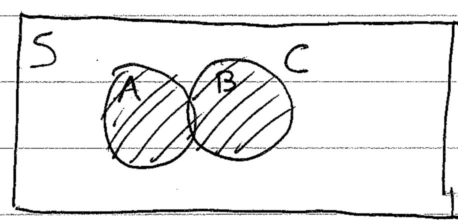
There also exists a set ϕ called the empty set which contains no elements.

<u>Note</u>: ϕ is a subset of every set.

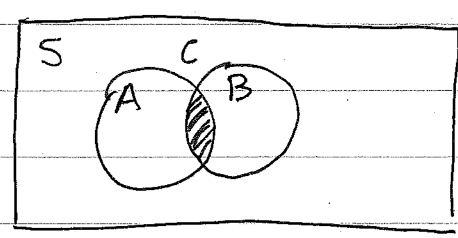
<u>Set Operations</u>:

<u>Equality</u>: $A = B$ $\quad (A \subset B$ and $B \subset A)$

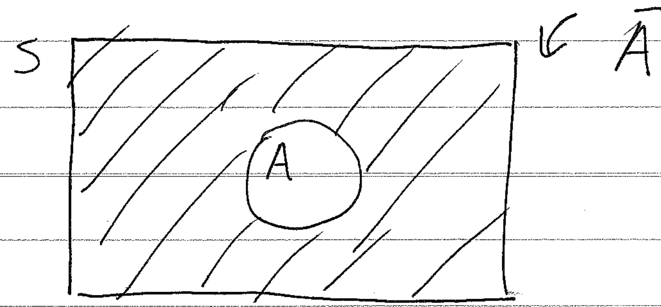
<u>Union</u>: $A \cup B = \{c \in S : c \in A$ or (inclusive) $c \in B\}$



<u>Intersections</u>: $A \cap B = \{c \in S : c \in A$ and $c \in B\}$



<u>Complement</u>: $\bar{A} = \{a' \in S : a' \notin A\}$



<u>Difference</u>: $A-B = \{a \in A : a \notin B\} = A \cap \bar{B}$